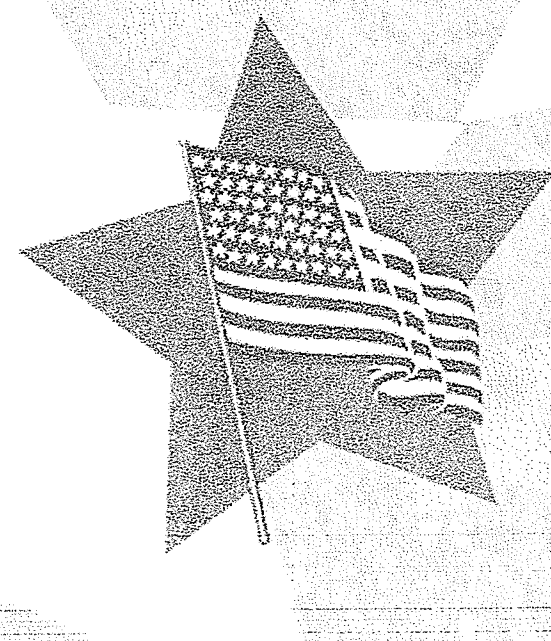
8th Grade Math Unit 1: Operations with Exponents



Worth County Middle School 2016-2017

1.1 Operations with Exponents

First let's start with a review of what exponents are. Recall that 3^4 means taking four 3's and multiplying them together. So we know that $3^4 = 3 \times 3 \times 3 \times 3 = 81$. You might also recall that in the number 3^4, three is called the base and four is called the exponent. Other reminds include that any number to the zero power is equal to one (so $5^0 = 1$) and any number is equal to itself to the first power (so $5^1 = 5$).

Sometimes it is easier to leave a number written as an exponent. For example, it is much easier to write 5^{20} instead of 95,367,431,640,625. Not only is sometimes simpler to write a number using exponents, but many operations are easier when the numbers are written as exponents.

Multiplying Numbers with the Same Base

Let's examine the problem $3^4 \times 3^4$ and write the answer as an exponent. Yes, we could multiply it out as a standard form number, $81 \times 81 = 6561$, but let's keep it in exponential form to see if it is any easier.

First, let's expand the problem: $3^4 \times 3^4 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$. Notice that the only operation that is happening here is multiplication and that we are multiplying the same number. That means we can say the following: $3^4 \times 3^4 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^8$. In short we see that $3^4 \times 3^4 = 3^8$. Do you see a rule that we could generalize from this?

Let's look at another example but this time with a variable.

$$y^7 \times y^4 = (y \times y \times y \times y \times y \times y \times y) \times (y \times y \times y \times y) = y^{11}$$

Can you find a rule that we can use when multiplying two exponent numbers with the same base? Yes, we can add the exponents. In other words, $z^5 \times z^6 = z^{5+6} = z^{11}$ would be a quicker way to show work for this problem. Generalizing this, we have the rule that $x^a \times x^b = x^{a+b}$.

Will this work with numbers without the same base? Let's find out by looking at $5^2 \times 2^3$. Many people think that $5^2 \times 2^3 = 10^5$, but we know that $5^2 \times 2^3 = 25 \times 8 = 200$ and that $10^5 = 100{,}000$. So we see that $5^2 \times 2^3 = 10^5$ is not true. Therefore we know that we can only add the exponents when we have the same base.

In fact, if asked to simplify $4^2 \times 7^2$ we would either have to multiply it out as a regular number or else leave it alone if we wanted it written using exponents.

Dividing Numbers with the Same Base

If multiplying numbers with the same base meant that we could add the exponents, what rule do you think we will discover when dividing numbers with the same exponent? Let's find out by looking at an example.

$$\frac{4^7}{4^5} = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4}$$

Note that since only multiplication and division is happening, five of fours in the denominator will "cancel" (they actually become one since four divided by four is one, we just call it "canceling") with five of the fours in the numerator. That means we get the following:

$$\frac{4^7}{4^5} = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4} = \frac{4 \times 4}{1} = 4^2$$

Let's look at one more example using variables before generalizing a rule for dividing exponent numbers with the same base.

$$\frac{q^6}{q^2} = \frac{q \times q \times q \times q \times q \times q}{q \times q} = \frac{q \times q \times q \times q}{1} = q^4$$

It looks like our rule is similar to the multiplication of exponent numbers with the same base, but this time we subtract the exponents. This gives us the general rule of $\frac{x^a}{x^b} = x^{a-b}$. For now we will only deal with division cases where the numerator exponent is larger than the denominator, but think ahead to what would happen if the denominator's exponent were larger. What do you think would happen?

A Power to a Power

We can also take exponents themselves to a power. For example, think of the problem $(2^3)^2$. Following our order of operations, we know that we have to do the parentheses first which means we get $(2^3)^2 = 8^2 = 64$. However, what if we wanted to leave our answer as a number to a power? Note the following:

$$(2^3)^2 = (2^3)(2^3) = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$$

Again, can you see a rule here? Let's look at an example with a variable to help again.

$$(g^4)^3 = (g^4)(g^4)(g^4) = (g \times g \times g \times g) \times (g \times g \times g \times g) \times (g \times g \times g \times g) = g^{12}$$

For a power to a power when using the same base we get the rule that you can multiply the exponents. This generalizes to $(x^a)^b = x^{ab}$.

Lesson 1.1

Perform the following operations leaving your answer as a number to a power. Remember that the parentheses can mean multiply as well.

1. $5^3 \times 5^7$

2. $(12^9)(12^0)$

3. $\frac{(t^5)(t^4)}{t^2}$

4. $\frac{4^{13}}{4^7} \times 4^{10}$

5. $\frac{f^5}{f}$

6. $\frac{u^{11}}{u^4}$

7. $(5^4)^5$

8. $(b^3)^6 \times (b^2)^9$

9. $(j^{11})^5$

Evaluate, meaning multiply out the exponents.

10. $3^2 \times 3^2$

11. $\frac{(2^{10})(2^2)}{2^9}$

12. $\frac{(5^3)^2}{5^4}$

13. $\frac{4^{12}}{4^{10}}$

14. $(5^3)^1 \times 5^0$

15. $(1^4)^2$

Determine if the following equations are true. Justify your answer.

16. $12^2 \times 12^7 = 12^6 \times 12^3$

17. $\frac{x^8}{x^3} = \frac{x^5}{x}$

18. $(t^5)^2 = (t^2)^5$

19. $(5^{10})^2 = (5^5)^5$

20. $\frac{6^0 \times 6^8}{6^4} = \frac{6^4}{6^0}$

21. $m^5 \times m^5 = (m^{10})^0$

22. $\frac{k^6}{k^2} = k^2 \times k^6$

23. $\frac{(7^4)^2}{7^3} = 7^3 \times 7^2$

24. $\frac{3 \times 3^4}{3^4} = (3^5)^1$

Determine the appropriate exponent to make the equation true.

25. $2^5 \times 2^{\boxed{?}} = 2^3 \times 2^3$

26. $\frac{p^6}{p^2} = \frac{p^7}{p^{\boxed{?}}}$

27. $(3^4)^3 = (3^6)^{\boxed{?}}$

28. $(5^{10})^2 = (5^{\boxed{?}})^5$

29. $\frac{b^2 \times b^8}{b^{\boxed{?}}} = \frac{b^7}{b^3}$

30. $9^{\boxed{?}} \times 9^8 = (9^3)^5$

31. $\frac{h^{\boxed{?}}}{h^2} = h^3 \times h^5$

32. $\frac{(6^{11})^{\boxed{?}}}{6^6} = 6^8 \times 6^8$

33. $\frac{3^{\boxed{?}} \times 3^9}{3^2} = (3^7)^1$

1.2 Negative Exponents

Last time we learned that when we divide exponent numbers with the same base we can subtract the exponents. We only examined problems where the numerator had a higher exponent than the denominator, but what would happen if the denominator had the higher exponent? Let's look.

$$\frac{5^3}{5^5} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5}$$

Notice that three of the fives will "cancel" (remember that they really become one because five divided by five is one). That means we are left with the following:

$$\frac{5^3}{5^5} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5 \times 5} = \frac{1}{5^2}$$

However, by following our rule from last time we know that we can also subtract the exponents which gives us:

$$\frac{5^3}{5^5} = 5^{-2}$$

Since $\frac{5^3}{5^5} = 5^{-2}$ and also $\frac{5^3}{5^5} = \frac{1}{5^2}$, by the transitive property we know that $5^{-2} = \frac{1}{5^2}$. We can now generalize this rule to say the following for any positive integer n:

$$x^{-n} = \frac{1}{x^n}$$

Negative Exponent as the Reciprocal

Another helpful way to think about negative exponents is as the reciprocal. Remember that the reciprocal of an integer is one over that integer because a number times its reciprocal must equal one. So 4^{-2} means the reciprocal of 4^2 which is $\frac{1}{4^2}$ or $\frac{1}{16}$. (Notice that $4^2 \times \frac{1}{4^2} = 1$ proving that we have the reciprocal.)

One last note is that except for scientific notation, we never leave negative exponents in a solution. We also take the reciprocal so that our exponent is positive. Let's look at a few more examples. Notice that we can evaluate the integer powers, but the variables to a power we have to leave the exponent.

$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ $\quad$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\quad$ $10^{-5} = \frac{1}{10^5} = \frac{1}{100{,}000}$ $\quad$ $13^{-1} = \frac{1}{13^1} = \frac{1}{13}$

$q^{-3} = \frac{1}{q^3}$ $\quad$ $w^{-7} = \frac{1}{w^7}$ $\quad$ $g^{-11} = \frac{1}{g^{11}}$ $\quad$ $j^{-1} = \frac{1}{j^1} = \frac{1}{j}$

Lesson 1.2

Evaluate the following negative exponents giving your answer as a fraction.

1. 5^{-3} 2. 2^{-2} 3. 3^{-2} 4. 7^{-2} 5. 4^{-3} 6. 10^{-3}

7. 10^{-2} 8. 1^{-14} 9. 6^{-2} 10. 2^{-4} 11. 9^{-1} 12. 5^{-2}

13. 10^{-4} 14. 8^{-1} 15. 3^{-4} 16. 6^{-1} 17. 4^{-2} 18. 11^{-1}

Simplify the negative exponents giving your answer as a fraction.

19. a^{-3} 20. b^{-2} 21. c^{-5} 22. d^{-6} 23. f^{-11} 24. g^{-13}

25. h^{-1} 26. j^{-4} 27. k^{-20} 28. m^{-9} 29. n^{-7} 30. p^{-10}

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Skills Practice

Powers and Exponents

Write each expression using exponents.

1. $2 \cdot 2 \cdot 2 \cdot 2$

2. $9 \cdot 9$

3. $7 \cdot 7 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

4. $\frac{3}{8} \cdot \frac{3}{8} \cdot \frac{3}{8}$

5. $c \cdot \frac{1}{4} \cdot c \cdot \frac{1}{4} \cdot \frac{1}{4}$

6. $s \cdot 6 \cdot s \cdot s \cdot 6 \cdot 6 \cdot s$

7. $8 \cdot x \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot 8$

8. $a \cdot (-4) \cdot b \cdot a \cdot b \cdot (-4) \cdot (-4)$

9. $\frac{1}{3} \cdot n \cdot 4 \cdot n \cdot \frac{1}{3} \cdot n \cdot 4 \cdot 4$

10. $9 \cdot 9 \cdot x \cdot w \cdot x \cdot y \cdot w \cdot 9 \cdot y$

Evaluate each expression.

11. 4^3

12. 2^5

13. $(-8)^3$

14. $\left(\frac{3}{5}\right)^4$

15. $2^8 - 3^2$

16. $2^3 \cdot 5^2$

17. $3^4 - (-4)^2$

18. $6 + 2^6$

19. $(-3)^3 \div 3^2$

ALGEBRA Evaluate each expression if $g = 2$ and $h = -3$.

20. g^4

21. $(g + ?)^3$

22. $?^4 - ?^3$

23. $g^3 + ?^2$

24. $(g - ?)^2 + ?^2$

25. $?^4 - (? - g)^3$

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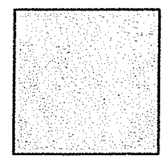
Problem-Solving Practice

Powers of Monomials

1. **DEBATE** Charmaine and Aaron are having a debate. Charmaine thinks the answer to their math homework is $(4^2)^4$, but Aaron says the answer is $(4^4)^2$. Explain how both Charmaine and Aaron can be correct.

2. **LAND** Kate was given a square plot of land in which to build. If one side of the plot was $(3a)^3$ feet long, express the area of her plot as a monomial.



$(3a)^3$

3. **CRAFTS** Numa loves beads and wants to know which amount would be more, a thousand beads or $(6^2)^3$ beads?

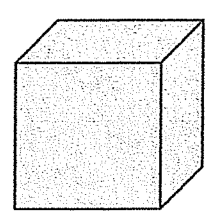
4. **TEST** The teacher marked Silvano's problem wrong on his test.

$$(4^5)^4 = 4^9$$

Explain what he did wrong and give the correct answer.

5. **WOOD** Dmitry calculated that he needs $6s^2$ square inches of wood for each crate he makes. Simplify the expression when s is replaced by t^4.

6. **VOLUME** Express the volume of the following cube as a monomial.



$(4d)^2$

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Homework Practice

Powers of Monomials

Simplify.

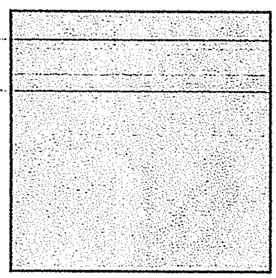
1. $(6t^5)^2$ 2. $(4w^9)^4$ 3. $(12k^6)^3$ 4. $(15m^8)^3$

5. $(4d^3e^5)^7$ 6. $(-4r^6s^{15})^4$ 7. $[(7^2)^2]^2$ 8. $[(3^2)^2]^3$

9. $(\frac{3}{5}a^6b^9)^2$ 10. $(4x^2)^3(3x^6)^4$ 11. $(0.6p^5)^3$ 12. $(\frac{1}{5}w^5x^3)^2$

GEOMETRY Express the area of each square below as a monomial.

13.



$9c^6d$

14.

$14g^5h^9$

15. **MEASUREMENT** In the Metric System, you would need to have $(10^4)^2$ grams to equal 1 metric ton. Simplify this measurement by multiplying the exponents, then simplify by finding the actual number of grams needed to equal 1 metric ton.

16. **GAMING** A video-game designer is using the expression $6n^3$ in a program to determine points earned, where n is the game level. Simplify the expression for the n^2 level.

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Skills Practice

Multiply and Divide Monomials

Simplify. Express using exponents.

1. $5^9 \circ 5^3$
2. $3^8 \circ 3$
3. $c \circ c^6$
4. $m^5 \circ m^2$
5. $3x \circ 4x^4$
6. $(2\square^7)(7\square)$
7. $-5d^6(8d^6)$
8. $(6k^5)(-k^4)$
9. $(-w)(-10w^3)$
10. $-7z^4(-3z^8)$
11. $bc^3(b^2c)$
12. $3a^4 \circ 6a^2$
13. $3m^3n^2(8mn^3)$
14. $7t^5(-6t^5)$
15. $(3ab^2)(a^2c^5)$
16. $(9p^4)(-8p^2)$
17. $\frac{2^9}{2^3}$
18. $\frac{3^8}{3^4}$
19. $\frac{5^9}{5^2}$
20. $\frac{8^7}{8}$
21. $\frac{b^{12}}{b^5}$
22. $\frac{12n^5}{4n^2}$
23. $\frac{14m^3}{7m^2}$
24. $\frac{9r^8}{3r^4}$
25. $\frac{24t^9}{6t^3}$
26. $\frac{18y^6}{2y}$
27. $\frac{a^4c^6}{a^2c}$
28. $\frac{5^{10}}{5^2}$

Simplify.

29. $\frac{4^8 \circ 5^3 \circ 7^6}{4^6 \circ 5^2 \circ 7^5}$.
30. $\frac{(-2)^9 \circ (-3)^7 \circ 4^3}{(-2)^5 \circ (-3)^5 \circ 4^1}$.
31. $\frac{3^{10} \circ (-6)^5}{3^7 \circ (-6)^2}$.
32. $\frac{9^8 \circ 10^{12}}{9^6 \circ 10^6}$.

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Problem-Solving Practice

Multiply and Divide Monomials

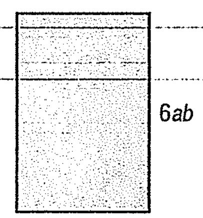
1. **SOUND** Decibels are units to measure sound. Ordinary conversation is rated at about 60 decibels (or a relative loudness of 10^6). Thunder is rated at about 120 decibels (or a relative loudness of 10^{12}). How many times greater is the relative loudness of thunder than the relative loudness of ordinary conversation?

2. **GEOMETRY** Express the area of a square with sides of length $5ab$ as a monomial.

3. **COMPUTERS** The byte is the fundamental unit of computer processing. The byte is based on powers of 2, as shown in the table. How many times greater is a gigabyte than a megabyte?

Memory Term	Number of Bytes
byte	2^0 or 1
kilobyte	2^{10}
megabyte	2^{20}
gigabyte	2^{30}

4. **GEOMETRY** The area of the rectangle in the figure is $24a^2b^3$ square units. Find the width of the rectangle.



5. **BOOKS** A publisher sells 10^6 copies of a new book. Each book has 10^2 pages. How many pages total are there in all of the books sold? Write the answer using exponents.

6. **RABBITS** Randall has 2^3 pairs of rabbits on his farm. Each pair of rabbits can be expected to produce 2^5 baby rabbits in a year. How many baby rabbits will there be on Randall's farm each year? Write the answer using exponents.

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1.3 Negative Exponents Operations

Now that we know negative exponents mean reciprocal, we can perform operations with negative exponents just like we did with positive exponents. Consider the following example of the multiplication rule. Notice that we still added the exponents, but just need to write our answer as a fraction if we have a negative exponent left after multiplication.

$$(5^3)(5^{-5}) = 5^{3+-5} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$(4^7)(4^{-5}) = 4^{7+-5} = 4^2 = 16$$

Now let's look at a division example. Remember that we found we can subtract the exponents as long as we have the same base.

$$\frac{5^2}{5^{-2}} = 5^{2--2} = 5^4 = 625$$

$$\frac{4^{-1}}{4^3} = 4^{-1-3} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$$

Finally we can see that the power to a power rule still works with negative exponents. We simply multiply the exponents.

$$(2^3)^{-2} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

$$(3^{-2})^{-2} = 3^4 = 81$$

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Lesson 1.3

Evaluate the following exponents operations giving your answer as a fraction where necessary.

1. $5^3 \times 5^{-4}$

2. $(12^9)(12^{-7})$

3. $\frac{(t^{-5})(t^4)}{t^2}$

4. $\frac{4^3}{4^{-7}} \times 4^{-10}$

5. $\frac{f^5}{f^{-1}}$

6. $(y^{-4})^{-5}$

7. $(2^3)^{-6} \times (2^2)^7$

8. $12^2 \times 12^{-4}$

9. $\frac{(k^{-3})^2}{k^4}$

10. $\frac{4^{-2}}{4}$

11. $(5^{-3})^2 \times 5^9$

12. $(0^{-4})^{10}$

Determine if the following equations are true. Justify your answer.

13. $12^{-2} \times 12^7 = 12^{-8} \times 12^3$

14. $\frac{x^{-5}}{x^{-3}} = \frac{x^5}{x^7}$

15. $(t^{-5})^2 = (t^{-2})^5$

16. $(5^{10})^2 = (5^{-5})^{-4}$

17. $\frac{6^{-6} \times 6^8}{6^4} = \frac{6^{-2}}{6^0}$

18. $m^7 \times m^7 = (m^{-7})^2$

19. $\frac{k^{-6}}{k^2} = k^2 \times k^{-10}$

20. $\frac{(7^{-4})^2}{7^3} = 7 \times 7^{12}$

21. $\frac{3 \times 3^4}{3^{10}} = (3^5)^{-1}$

Determine the appropriate exponent to make the equation true.

22. $2^5 \times 2^{\square} = 2^{-6} \times 2^3$

23. $\frac{p^6}{p^{-2}} = \frac{p^{\square}}{p^2}$

24. $(3^{-4})^3 = (3^{-2})^{\square}$

25. $(5^{12})^{-2} = (5^3)^{\square}$

26. $\frac{b^{-2} \times b^8}{b^5} = \frac{b^{\square}}{b^3}$

27. $9^2 \times 9^{-8} = (9^{\square})^3$

28. $\frac{h^{-2}}{h^{\square}} = h^3 \times h^{-5}$

29. $\frac{(6^2)^{\square}}{6^6} = 6^{-8} \times 6^8$

30. $\frac{3^{-4}}{3^{\square} \times 3^9} = (3^7)^{-1}$

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Problem-Solving Practice

Negative Exponents

1. MOTHS A Polyphemus Moth caterpillar weighs about $\frac{1}{64^2}$ times less when it first becomes a larva than it does when it is fully grown. Write this number using a negative exponent.

2. WEIGHT The length of one common termite is about 30^{-2} meters. Write this number using a positive exponent.

3. MONEY The school system spent 3^8 dollars on fuel for buses and school vehicles per week last year. This year, they spent 3^{10} dollars per week. How many times more did they spend per week this year than last year?

4. MEASUREMENT The table converts the size of each measurement to kilograms. Write each number using a positive exponent.

Amount	Amount in Kilograms
1 centigram	10^{-5}
1 decigram	10^{-4}
1 dekagram	10^{-2}

5. SCIENCE Electrons are smaller than 10^{-18} meters. Write this number using a positive exponent.

6. MONEY A bank loans a new business 6^7 dollars to get started. If the business pays back 6^5 dollars per year, how many years will it take to pay off the loan? Write your answer using a positive exponent.

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1.5 Scientific Notation Operations

Now that we understand what scientific notation is, we can begin performing operations with these numbers just as scientists have to in their research.

Multiplication and Division with Scientific Notation

Let's say the debt in the United States is now near 1.8×10^{13} dollars and that there are about 300,000,000 people in the United States. If every person paid their fair share of the debt, approximately how much would that be per person? To solve this problem we need to divide the debt between all the people. Let's start by writing the standard form number in scientific notation.

$$300{,}000{,}000 = 3 \times 10^{8}$$

Now we can begin to divide those scientific notation numbers by writing the problem in fraction form:

$$\frac{1.8 \times 10^{13}}{3 \times 10^{8}}$$

How do we actually divide these? Since the only operations happening are multiplication and division, we can divide the whole numbers and then divide powers of ten. Why does this work? Because we can actually split this into two different fractions and simply each fraction as follows:

$$\frac{1.8 \times 10^{13}}{3 \times 10^{8}} = \frac{1.8}{3} \times \frac{10^{13}}{10^{8}} = 0.6 \times 10^{5}$$

The problem now is that our answer is not in scientific notation. We need to be a single non-zero digit before the decimal. We could multiply the six tenths by ten to make it six, but remember with any expression we can't just change values. The only thing we can multiply or divide by is one. So if multiply by ten, we have to divide by ten so that there is no change in the actual value of the number, like so:

$$0.6 \times 10^{5} = 0.6 \times (10 \div 10) \times 10^{5} = (0.6 \times 10) \times (10^{5} \div 10) = 6 \times 10^{4}$$

While this appears confusing, it really just keeping everything balanced. Since we made the 0.6 bigger by a factor of 10, we have to make the 10^{5} smaller by a factor of 10 to balance it.

Let's look at another example. There are approximately 6×10^{9} people in the world. If each person made 2.5×10^{4} dollars in a year, how much money was made worldwide? In this case we need to multiply the numbers, rearranging using the commutative property as follows:

$$(6 \times 10^{9})(2.5 \times 10^{4}) = 6 \times 2.5 \times 10^{9} \times 10^{4} = 15 \times 10^{13}$$

Again, we need to get our answer in scientific notation. In this case, the 15 needs to be a 1.5, meaning it needs to be smaller a single factor of 10. Therefore we will have to make the 10^{13} bigger by a factor of ten making it 10^{14}. This means our final answer is 1.5×10^{14} dollars produced in that year.

Addition and Subtraction with Scientific Notation

When adding or subtracting numbers in scientific notation, we have to remember that place value matters. For example when adding 123 and 56, we have to add the 6 and 3 because they are both in the ones place. In the same way, we have to add the 2 and 5 because they are both in the tens place.

Since scientific notation makes every number a single digit followed by a decimal and a power of ten, the place value gets hidden. This means that we can only add or subtract scientific notation numbers if they have the same power of ten (since the power of ten controls the place value). If they don't have the same power of ten, we will have to rewrite one of the numbers in a way such that the powers of ten are equal.

For example, let's solve the problem $3.7 \times 10^6 + 4.3 \times 10^5$ where the powers of ten are different. First we need to make the powers of ten be the same. Since 3.7×10^6 is the larger number, we'll leave that alone and convert 4.3×10^5 into an equal number that has $\times 10^6$ at the end. Since we want the power of ten to be bigger by a single factor of ten, we'll need to make the 4.3 smaller by a factor of ten as follows:

$$4.3 \times 10^5 = 0.43 \times 10^6$$

Now that we have the same power of ten (which means the same place value), we can solve as follows:

$$3.7 \times 10^6 + 0.43 \times 10^6 = 4.13 \times 10^6$$

Notice that we simply added the 3.7 and the 0.43 together. Notice that we could have turned the scientific notation numbers into standard form numbers and then added. However, this is only convenient with small powers of ten.

$$3.7 \times 10^6 = 3{,}700{,}000$$

$$4.3 \times 10^5 = 430{,}000$$

$$3.7 \times 10^6 + 4.3 \times 10^5 = 3{,}700{,}000 + 430{,}000 = 4{,}130{,}000 = 4.13 \times 10^6$$

Estimating Very Large Numbers with Powers of 10

Scientists have measured the temperature at the edge of the sun to be around 5,400° C. Let's round that number to a single digit times a power of ten. Looking at the leftmost digit, which is 5, should that stay a 5 or round up to 6? Since the next number is only a four, it will stay a 5. That means $5{,}4000 \approx 5{,}000$. Now let's look at our place value.

This shows us that we can write 5,000 as 5×10^3. We may be tempted to simply count the number of zeros and use that as the power, but that will only work for this specific type of problem. Instead we should continue to think about place value.

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Let's look at one final example of rounding a large number to a single digit times a power of ten. In a penny there may be approximately 19,370,000,000,000,000,000,000 atoms.

$$19{,}370{,}000{,}000{,}000{,}000{,}000{,}000 \approx 20{,}000{,}000{,}000{,}000{,}000{,}000{,}000$$

$$20{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 2 \times 10^{22}$$

$$19{,}370{,}000{,}000{,}000{,}000{,}000{,}000 \approx 2 \times 10^{22}$$

So there are approximately 2×10^{22} atoms in a penny. Notice that we could simply count the place values from right to left starting from the where the decimal would be to find our exponent. In other words, we could count how many places the decimal "moved" to get the new number.

Estimating Very Small Numbers with Powers of 10

This will work the same way except that now we will have negative powers of ten since we will be dealing with very small decimals. For example, a single atom in that penny is approximately 0.0000000312 cm across. We can round that using powers of ten in nearly the same way.

First, look at the leftmost non-zero digit, which is a 3. Should that stay a 3 or should it round up to a 4? It should stay a 3 because the next digit is a one and we only round up when the number is five or greater. That means that $0.0000000312 \approx 0.00000003$. Now let's examine that in our place value chart. Notice that we have negative exponents because the tenths place is really the fraction $\frac{1}{10}$ which is 10^{-1} and so forth.

Now we see that $0.0000000312 \approx 3 \times 10^{-8}$.

Let's do one final example of rounding a small number using a single digit times a power of ten. Round the number 0.000 000 000 000 000 871 to single digit times a power of ten. (Notice that sometimes we put spaces between every three zeros to make it easier to count how many zeros are there.)

$$0.000\ 000\ 000\ 000\ 000\ 871 \approx 0.000\ 000\ 000\ 000\ 000\ 9$$

$$0.000\ 000\ 000\ 000\ 000\ 9 = 9 \times 10^{-16}$$

$$0.000\ 000\ 000\ 000\ 000\ 871 \approx 9 \times 10^{-16}$$

Notice again that we could simply count the number of places the decimal "moved" to make it 9. That took 16 places for the decimal to move; therefore we will use the negative 16th power as our exponent.

Estimating Operations

When faced with an operation involving scientific notation, we can estimate the final solution by rounding the numbers first then performing the operation. This will make your final solution an estimate as well.

Lesson 1.5

Compute the EXACT answer to each of the following questions giving your answer in scientific notation.

1. $(3 \times 10^{-6})(3 \times 10^{9})$

2. $\frac{6.8 \times 10^{9}}{2 \times 10^{5}}$

3. $4.5 \times 10^{7} + 41{,}000{,}000$

4. $8.4 \times 10^{7} - 3.1 \times 10^{7}$

5. $(2.4 \times 10^{4})(3{,}000)$

6. $\frac{5.4 \times 10^{8}}{3{,}000}$

7. $3.9 \times 10^{13} + 4.2 \times 10^{13}$

8. $8.2 \times 10^{-5} - 0.000\,059$

9. $(1.3 \times 10^{-4})(4.2 \times 10^{11})$

10. $\frac{4.5 \times 10^{9}}{1.5 \times 10^{13}}$

11. $1.3 \times 10^{7} + 4 \times 10^{7}$

12. $5.2 \times 10^{7} - 12{,}000{,}000$

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ESTIMATE the answer to each of the following questions giving your answer as single digit times a power of ten. (5 pts; 2 pts for correct digits, 3 pts for correct power of ten)

13. How many times bigger is the distance from Earth to the sun of 9.3×10^6 miles than the furthest distance from Earth to the moon of 3×10^5 miles?

---

14. The temperature halfway to the Sun from Mercury is approximately 1,800° C and scientists theorize that it may be up to 26,000 times hotter at the center of the Sun. Approximately how hot is it at the center of the Sun?

15. Each shrimp weighs approximately 0.000 27 g and a shrimp company can bring in over 3,100,000,000 shrimp per year. Approximately how much would that many shrimp weigh?

16. The Earth has a mass of about 1×10^{25} kg. Neptune has a mass of 1.8×10^{27} kg. How many times bigger is Neptune than Earth?

17. A country has an area of approximately 8,400,000,000 square miles and has approximately 210,000 people. How much area is this per person?

18. A blue whale can eat 300,000,000 krill in a day. All of that krill weighs approximately 6,300,000,000 mg. About how much does each krill weigh?

19. The US spends on average 10,200 dollars on each student per year. There are about 77,000,000 students in the United States. How about much money total is spent on students each year?

20. McDonald's has about 210,000 managers and each makes on average 39,000 dollars per year. How much money does McDonald's spend on managers each year?

NAME ______________________ DATE ____________ PERIOD ________

Skills Practice

Scientific Notation

Write each number in standard form.

1. 6.7×10^1
2. 6.1×10^4
3. 1.6×10^3
4. 3.46×10^2
5. 2.91×10^5
6. 8.651×10^7
7. 3.35×10^{-1}
8. 7.3×10^{-6}
9. 1.49×10^{-7}
10. 4.0027×10^{-4}
11. 5.2277×10^{-3}
12. 8.50284×10^{-2}

Write each number in scientific notation.

13. 34
14. 273
15. 79,700
16. 6,590
17. 4,733,800
18. 2,204,000,000
19. 0.00916
20. 0.29
21. 0.00000571
22. 0.0008331
23. 0.0121
24. 0.00000018

NAME ______________________ DATE ____________ PERIOD ________

Problem-Solving Practice

Scientific Notation

1. MEASUREMENT There are about 25.4 millimeters in one inch. Write this number in scientific notation.	2. POPULATION In the year 2000, the population of Rahway, New Jersey, was 26,500. Write this number in scientific notation.
3. MEASUREMENT One nanometer is 1.0×10^{-5} meter. Write this number in standard notation.	4. PHYSICS The speed of light is about 1.86×10^{5} miles per second. Write this number in standard notation.
5. COMPUTERS A CD can store about 650,000,000 bytes of data. Write this number in scientific notation.	6. SPACE The diameter of the Sun is about 1.39×10^{9} meters. Write this number in standard notation.
7. BIOLOGY The diameter of a certain virus is 0.000000028 meter. Write this number in scientific notation.	8. MASS The mass of planet Earth is about 5.98×10^{24} kilograms. Write this number in standard notation.

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NAME ______________________ DATE ______________ PERIOD _________

Skills Practice

Compute with Scientific Notation

Evaluate each expression. Express the result in scientific notation.

1. $(5.8 \times 10^5)(6.4 \times 10^2)$

2. $(3.92 \times 10^6)(2.2 \times 10^4)$

3. $\frac{2.952 \times 10^6}{3.6 \times 10^3}$

4. $\frac{2.052 \times 10^7}{5.4 \times 10^4}$

5. $(6.9 \times 10^7) + (2.12 \times 10^5)$

6. $(1.78 \times 10^4) + (5.35 \times 10^3)$

7. $(8.4 \times 10^7) - (6.3 \times 10^6)$

8. $(9.62 \times 10^5) - (2.58 \times 10^3)$

9. $\frac{6.256 \times 10^8}{6.8 \times 10^4}$

10. $\frac{2.888 \times 10^5}{7.22 \times 10^2}$

11. $(3.68 \times 10^3)(2.4 \times 10^6)$

12. $(7.2 \times 10^7)(1.82 \times 10^2)$

13. $(6.78 \times 10^4) - (4.13 \times 10^2)$

14. $\frac{3.024 \times 10^6}{4.8 \times 10^2}$

15. $(5.9 \times 10^8) + (2.6 \times 10^6)$

16. $(3.45 \times 10^7)(1.68 \times 10^4)$

17. $(8.33 \times 10^3) + (4.1 \times 10^5)$

18. $(6.82 \times 10^5) - (3.11 \times 10^4)$

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NAME ______________________ DATE ____________ PERIOD ________

Problem-Solving Practice

Compute with Scientific Notation

1. OCEAN Humpback whales are known to weigh as much as 8×10^4 pounds. The tiny krill they eat weigh only 2.1875×10^{-3} pounds. How many times greater than krill are humpback whales?

2. MEASUREMENT One inch is equal to 1.5782×10^{-5} miles. One centimeter is equal to 6.2137×10^{-6} miles. How many miles greater is one inch than one centimeter?

3. MONUMENT The Statue of Liberty is about 1.5108×10^2 feet tall from the base to the torch. The pedestal is 1.54×10^2 feet tall. How tall is the Statue of Liberty from the foundation of the pedestal to the top of the torch?

4. FUNDRAISER The table shows the amount of money raised by each region for cancer awareness. How much money did the North and South raise together?

Region	Amount Raised ($)
East	1.46×10^4
North	2.38×10^4
South	6.75×10^3
West	8.65×10^3

5. TURKEYS When the National Wild Turkey Federation was formed in 1973, there were only about 1.3×10^6 wild turkeys in North America. Now there are over 7×10^6 wild turkeys in North America. About how many more turkeys are there now than there were in 1973?

6. MONEY A bank starts the day with 2.93×10^4 dollars in the vault. At the end of the day, the bank has 3.5×10^5 dollars in the vault. How much more money is in the vault at the end of the day than there was in the morning?

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Review Unit 1: Exponents

No calculator necessary. Please do not use a calculator.

Unit 1 Goals

- *Know and apply the properties of integer exponents to generate equivalent numerical expressions. (8.EE.1)*
- *Use numbers expressed in the form of a single digit times an integer power of* 10 *to estimate very large or very small quantities, and to express how many times as much one is than the other. (8.EE.3)*
- *Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology. (8.EE.4)*

Evaluate, meaning multiply out the exponent, giving your answer as a fraction when necessary.

1. $4^4 \times 4^{-2}$
2. $(3^2)(3^{-5})$
3. $(6^2)^0$
4. $\frac{3^4}{3^9}$
5. $(v^6)^{-3}$
6. $\frac{b^8}{b^{-2}}$
7. $(k^{-10})(k^{-2})$
8. $(5^{-3})^{-1}$
9. $\frac{2^{-6}\times 2^3}{2^{-5}}$
10. $(j^3)^{-2} \times j^6$
11. $\frac{(m^{-3})^5}{m^5}$
12. $\frac{4^6}{4^{-2}} \times 4^{-6}$

Determine if the following equations are true. Justify your answer.

13. $8^{-5} \times 8^6 = 8^0 \times 8$
14. $(j^2)^{-5} = \frac{j^{10}}{j^2}$
15. $\frac{m^3 \times m^{-5}}{m^2} = \frac{m^4}{m^0}$
16. $(4^{-5})^4 = (4^{10})^{-2}$

Determine the appropriate exponent to make the equation true.

17. $\frac{p^5}{p^{-5}} = (p^2)^{?}$
18. $2^{-10} \times 2^2 = 2^{-4} \times 2^{?}$
19. $\frac{f^0}{f^{-7}} = \frac{f^{14}}{f^{?}}$
20. $\frac{(9^2)^4}{9^{-2}} = (9^{?})^2$

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Write the following numbers in scientific notation.

21. 9,089,000,000

22. 810,000,000,000

23. 0.000 000 27

24. 0.001 06

Write the following numbers in standard form.

25. 5.14×10^{-6}

26. 4.07×10^{9}

27. 3.1×10^{-7}

28. 7.109×10^{4}

Choose the best unit of measurement for the following problems.

29. Would the weight of a strand of hair, about 6.9×10^{-5} grams, be best expressed using tons, pounds, or milligrams?

30. Would the area of the City of Charleston, about 24.9×10^{11} square centimeters, be best expressed using square kilometers, square meters, or square millimeters?

Estimate each of the following as a single digit times a power of ten. Then compute each of the following giving your answer in scientific notation.

31. $(2.1 \times 10^{6})(500)$

32. $\frac{6.3\times10^{7}}{3\times10^{3}}$

33. $4.2 \times 10^{7} - 3,000,000$

34. $6.2 \times 10^{-6} + 0.000\ 005$

35. $\frac{4.8\times10^{3}}{4\times10^{6}}$

36. $(3 \times 10^{-9})(2,300)$

Answer the following questions giving <u>both</u> an estimated answer (single digit times a power of ten) and a precise answer (scientific notation).

37. In the United States, there are approximately 300,000,000 people that use a total of approximately 360,000,000,000 gallons of water in a day. How much water does each person use?

38. A football field has an area of 56,560 ft^2 and a tennis court 2,800 ft^2. How many times bigger in area is a football field than a tennis court?

39. Three thousand people sued McDonald's for hot coffee and were compensated at least $280,000 for each person. How much money was distributed in total?

40. There were 9,700,000 Macintosh computers sold in 2008. If Macintosh earns an average of $200 profit per computer sold, how much profit did the company make selling computers in 2008?

7.1 Identifying Irrational Numbers

In previous years you studied rational numbers. Recall that rational numbers are any number that can be expressed as a fraction where the numerator and denominator are both integers. Sometimes you will see this fraction written as $\frac{p}{q}$ where p and q are both integers.

You might also recall that this means that repeating decimals, meaning decimals that follow a repeating numeric pattern as some point are both rational numbers. For example, $0.08\overline{3}$ and $0.\overline{142857}$ are rational numbers because they are a repeating pattern. In fact, $0.08\overline{3} = \frac{1}{12}$ and $0.\overline{142857} = \frac{1}{7}$. Remember that even terminating decimals, meaning decimals that stop, are really repeating decimals and therefore rational. For example, $0.75 = 0.75000000000000\ldots$ which shows that the zero is repeating meaning 0.75 is rational. In fact, it is equal to $\frac{3}{4}$.

All of this helps us to define irrational numbers. Recall that the prefix *ir-* means "not" so that we can define irrational numbers as numbers that are not rational. In other words, an irrational number cannot be written as a fraction. An irrational number written as a decimal would go forever and have no repeating pattern.

The most common example of this is the number π which you may know is approximately 3.14 or $\frac{22}{7}$. However, both of those values are only rational estimates of π. Other than a few special numbers like π or *e* (which you'll learn about in later math courses), irrational numbers come up most often when dealing with square roots.

Recall that a square root is the inverse operation of squaring a number. In other words, we are asking ourselves, "What number multiplied by itself will equal the given number?" The symbol for square root is $\sqrt{\ }$ and you should remember some basics such as $\sqrt{25} = 5$ or $\sqrt{0.49} = 0.7$ when we take the principal (or positive) square root.

When square roots don't have exact solutions such as the examples above, they are irrational. So all of the following are irrational numbers because they don't have an exact solution: $\sqrt{60}, \sqrt{11}, \sqrt{2}, \sqrt{77}, \sqrt{21}$. In particular, note that $\sqrt{2}$ is irrational. This is something you should have memorized.

Using Technology

Most calculators have a square root button, but that will not necessarily tell you for sure whether a number is rational or not as you may not be able to plug in the whole number you want to find the square root of due to the display screen size. Also, the calculator may give you an answer such as 0.1344567927 which looks like it could be irrational since we can't see a pattern, but the number $0.\overline{1344567927}$ is rational and we don't know for sure if the pattern repeats because the calculator did not return enough numbers. Rely on your brain and not the calculator to determine if a number is rational or irrational.

Lesson 7.1

Identify which of the following numbers are rational or irrational and explain how you know.

1. $\sqrt{25}$ 2. $\sqrt{24}$ 3. $-\sqrt{36}$ 4. $-\sqrt{64}$ 5. $-\sqrt{27}$ 6. $\frac{3}{8}$

7. 0.45 8. $0.\overline{2}$ 9. $\sqrt{49}$ 10. $\sqrt{18}$ 11. $-\sqrt{10}$ 12. $\frac{11}{21}$

13. $\frac{2}{13}$ 14. $0.\overline{42}$ 15. 0.39 16. $-\sqrt{100}$ 17. $-\sqrt{16}$ 18. $-\sqrt{43}$

19. ~~If the number 0.77 is displayed on a calculator that can only display ten digits,~~ do we know whether it is rational or irrational? In one complete sentence explain why.

20. If the number 0.123456789 is displayed on a calculator that can only display ten digits, do we know whether it is rational or irrational? In one complete sentence explain why.

21. If the number 0.987098709 is displayed on a calculator that can only display ten digits, do we know whether it is rational or irrational? In one complete sentence explain why.

22. If the number 0.425364758 is displayed on a calculator that can only display ten digits, do we know whether it is rational or irrational? In one complete sentence explain why.

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7.4 Comparing and Ordering Irrational Numbers on a Number Line

To compare irrational numbers that are square roots, we can simply examine the number that we are taking the square root of. For example, we know that $\sqrt{15} < \sqrt{17}$ because 15 is less than 17.

However, when we compare irrational numbers such as $\sqrt{10}$ and π, it is simplest to compare rational approximations of each written as a decimal. We know that $\sqrt{10} \approx 3.16$ and that $\pi \approx 3.14$. Therefore we can say that $\sqrt{10} > \pi$. Notice that it was useful to approximate the irrational numbers to two decimal places in this case even though it wasn't entirely necessary.

The same is true for comparing irrational and rational numbers. By finding a rational approximation of the irrational numbers, we can compare values such as π and $\frac{22}{7}$. For these numbers we may have to go to a three decimal places for our approximation and the use of a calculator would make sense. Rounded to three decimal places, we find that $\pi \approx 3.142$ and $\frac{22}{7} \approx 3.143$ which means that $\pi < \frac{22}{7}$.

Once we know how to compare two numbers, we can then order a set of numbers through comparison of two numbers at a time. For example, we could list from least to greatest $\sqrt{10}, \pi$, 3.14, and $\frac{22}{7}$. We know the following:

$$\sqrt{10} \approx 3.162$$

$$\pi \approx 3.142$$

$$\frac{22}{7} \approx 3.143$$

We see that $\sqrt{10}$ is greater than all the other numbers given. We also note that 3.14 is the smallest because it is equal to 3.140 to three decimal places. Therefore we can list them in order like so: $3.14, \pi, \frac{22}{7}, \sqrt{10}$. Notice that the closer the numbers are to each other, the more decimal places of accuracy we need in our rational approximation.

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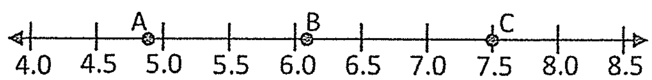
Locating Irrational Numbers on a Number Line

Again, rational approximations of irrational numbers will be our friend. On a number line, we generally list rational number markers. On the simplest number lines, we count by integers. On a standard English ruler, we count by fractions, usually $\frac{1}{16}$ inch or $\frac{1}{8}$ inch. On a standard metric ruler, we count by millimeters which are each .1 centimeters. No matter how the number line is set up, we will still need the rational approximations of the irrational numbers.

For example, let's try to place the following irrational numbers on the number line: $\sqrt{37}$, $\sqrt{42}$, and $\sqrt{24}$. First we will make a quick, one decimal place approximation of each. $\sqrt{37} \approx 6.1$ since 37 is just over 36, $\sqrt{56} \approx 7.5$ since 56 is about half-way between the perfect squares of 49 and 64, and $\sqrt{24} \approx 4.9$ since 24 is just under 25. Now examine where the dots are located on the following number line.



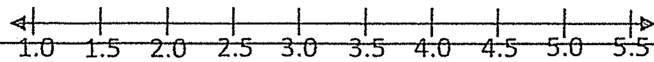
Note that point A must be $\sqrt{24}$ since it is just under 5, point B must be $\sqrt{37}$ since it is just over 6, and point C must be $\sqrt{56}$ since it is right at 7.5 on the number line.

In the same way that you can identify which point on a number line goes with which irrational number, you can also place points on a number line to represent the irrational number.

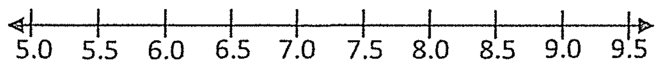
Lesson 7.4

Place a point on the number line given for each of the following irrational numbers.

1. Point A: $\sqrt{2}$
2. Point B: $\sqrt{17}$
3. Point C: $\sqrt{11}$
4. Point D: $\sqrt{8}$
5. Point E: $\sqrt{5}$

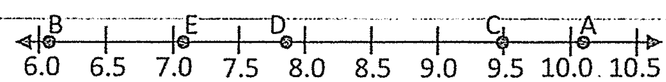


6. Point V: $\sqrt{26}$
7. Point W: $\sqrt{88}$
8. Point X: $\sqrt{77}$
9. Point Y: $\sqrt{37}$
10. Point Z: $\sqrt{30}$

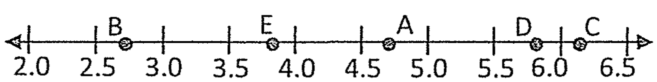


Name the point on the number line associated with each irrational number.

11. $\sqrt{50}$
12. $\sqrt{103}$
13. $\sqrt{62}$
14. $\sqrt{90}$
15. $\sqrt{37}$



16. $\sqrt{7}$
17. $\sqrt{22}$
18. $\sqrt{34}$
19. $\sqrt{38}$
20. $\sqrt{15}$



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Compare the following numbers using < or >.

21. $\sqrt{32}\ \square\ 5.1$

22. $\sqrt{38}\ \square\ \sqrt{42}$

23. $\sqrt{17}\ \square\ \frac{9}{2}$

24. $\sqrt{49}\ \square\ 7.1$

25. $\sqrt{99}\ \square\ \frac{28}{3}$

26. $\sqrt{17}\ \square\ 4.5$

27. $\frac{43}{5}\ \square\ \sqrt{65}$

28. $\sqrt{12}\ \square\ \sqrt{21}$

29. $\sqrt{16}\ \square\ 3.9$

30. $\sqrt{2}\ \square\ \frac{7}{4}$

31. $\sqrt{50}\ \square\ \frac{15}{2}$

32. $\sqrt{9}\ \square\ 3.01$

List the following numbers in order from least to greatest.

33. $\sqrt{16}$, 4.2, $\frac{39}{8}$

34. $\sqrt{24}$, $\sqrt{33}$,5.1

35. $\sqrt{100}$, $\sqrt{110}$, $\frac{32}{7}$

36. 9.4 , $\frac{19}{2}$, $\sqrt{80}$

37. $\sqrt{35}$, $\sqrt{32}$, $\sqrt{37}$, $\frac{22}{3}$

38. $\sqrt{10}$, 3.5, $\sqrt{15}$, $\frac{13}{3}$

39. $\sqrt{65}$, $\sqrt{60}$, 8.5, $\frac{37}{4}$

40. $\sqrt{39}$, $\sqrt{25}$, 5.3, $\sqrt{26}$, $\frac{23}{4}$

41. $\sqrt{12}$, $\sqrt{15}$, 4.3, $\sqrt{9}$, $\frac{14}{5}$

42. $\sqrt{49}$, $\sqrt{63}$, 7.3, $\sqrt{38}$, $\frac{15}{2}$

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