8th Grade Mathematics

Unit 1

Exponents

Worth County Middle School

2015-2016

Unit 1: Exponents

Unit EQ: How can I use integer exponents and radicals to solve real-life problems?

Integer Exponents	Scientific Notation	Irrational Numbers	Square/Cube Roots
Essential Questions: • How can I apply the properties of integer exponents to generate equivalent numerical expressions? • How do I simplify and evaluate numeric expressions involving integer exponents?	**Essential Questions:** • How can I represent very small and large numbers using integer exponents and scientific notation? • How can I perform operations with numbers expressed in scientific notation? • How can the properties of exponents and knowledge of working with scientific notation help me interpret information?	**Essential Questions:** • What is the difference between rational and irrational numbers? • What strategies can I use to create and solve linear equations with one solution, infinitely many solutions, or no solutions?	**Essential Questions:** • Why is it useful for me to know the square root of a number? • How do we locate approximate locations of irrational numbers on a number line and estimate the values of irrational numbers? • Why do we approximate irrational numbers?
GA Standards of Excellence: MGSE8.EE.1	***GA Standards of Excellence:*** MGSE8.EE.3, MGSE8.EE.4	***GA Standards of Excellence:*** MGSE8.EE.7, MGSE8.EE.7a, MGSE8.EE.7b, MGSE8.NS.1, MGSE8.NS.2	***GA Standards of Excellence:*** MGSE8.EE.2, MGSE8.NS.2
Vocabulary: Exponent Exponential Notation	**Vocabulary:** Decimal Expansion Scientific Notation Significant Digits	**Vocabulary:** Addition Property of Equality Multiplication Property of Equality Linear Equation *Rational Variable *Irrational Multiplicative Inverses Additive Inverses *Solution Algebraic Expression Equation *Like Terms Evaluate *Inverse Operation	**Vocabulary:** Cube Root Square Root Perfect Square Radical

Unit 1 Georgia Standards of Excellence

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that x2 = p (where p is a positive rational number and lxl < 25) has 2 solutions and x3 = p (where p is a negative or positive rational number and lxl < 10) has one solution. Evaluate square roots of perfect squares < 625 and cube roots of perfect cubes > -1000 and < 1000.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

MGSE8.EE.7 Solve linear equations in one variable.
MGSE8.EE.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

MGSE8.EE.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

MGSE8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

MGSE8.NS.2 Use rational approximation of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., estimate π^2 to the nearest tenth). For example, by truncating the decimal expansion of √2 (square root of 2), show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Lesson 1:

PROPERTIES OF EXPONENTS

$$x^3 = x \cdot x \cdot x$$

There are a couple of operations you can do on powers and we will introduce them now.

We can multiply powers with the same base

$$x^4 \cdot x^2 = (x \cdot x \cdot x \cdot x) \cdot (x \cdot x) = x^6$$

This is an example of the product of powers property tells us that when you multiply powers with the same base you just have to add the exponents.

$$x^a \cdot x^b = x^{a+b}$$

We can raise a power to a power

$$\left(x^2\right)^4 = (x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x) = x^8$$

This is called the power of a power property and says that to find a power of a power you just have to multiply the exponents.

When you raise a product to a power you raise each factor with a power

$$(xy)^2 = (xy) \cdot (xy) = (x \cdot x) \cdot (y \cdot y) = x^2y^2$$

This is called the power of a product property

$$(xy)^a = x^a y^a$$

As well as we could multiply powers we can divide powers.

$$\frac{x^4}{x^2} = \frac{x \cdot x \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x}} = x^2$$

This is an example of the quotient of powers property and tells us that when you divide powers with the same base you just have to subtract the exponents.

$$\frac{x^a}{x^b} = x^{a-b}, \quad x \neq 0$$

When you raise a quotient to a power you raise both the numerator and the denominator to the power.

$$\left(\frac{x}{y}\right)^2 = \frac{x}{y} \cdot \frac{x}{y} = \frac{x \cdot x}{y \cdot y} = \frac{x^2}{y^2}$$

This is called the power of a quotient power

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}, \quad y \neq 0$$

When you raise a number to a zero power you'll always get 1.

$$1 = \frac{x^a}{x^a} = x^{a-a} = x^0$$

$$x^0 = 1, \quad x \neq 0$$

Negative exponents are the reciprocals of the positive exponents.

$$x^{-a} = \frac{1}{x^a}, \quad x \neq 0$$

$$x^a = \frac{1}{x^{-a}}, \quad x \neq 0$$

NAME ______________________

Check What You Know

Integers and Exponents

CHAPTER 1 PRETEST

Find the value of each expression.

	a	b	c
1.	$7^3 =$ ______	$8^5 =$ ______	$4^2 =$ ______
2.	$9^4 =$ ______	$1^5 =$ ______	$6^8 =$ ______
3.	$4^{-3} =$ ______	$3^{-5} =$ ______	$7^{-4} =$ ______
4.	$2^{-5} =$ ______	$9^{-3} =$ ______	$10^{-3} =$ ______
5.	$7^4 =$ ______	$3^{-4} =$ ______	$5^9 =$ ______

Rewrite each multiplication or division expression using a base and an exponent.

6.	$4^5 \div 4^2 =$ ______	$6^{-5} \times 6^3 =$ ______	$8^{-4} \div 8^{-2} =$ ______
7.	$9^{11} \div 9^6 =$ ______	$5^{-3} \times 5^{-1} =$ ______	$3^{-6} \div 3^4 =$ ______
8.	$8^2 \times 8^3 =$ ______	$6^4 \times 6^7 =$ ______	$4^{-2} \div 4^{-5} =$ ______
9.	$7^6 \div 7^3 =$ ______	$4^8 \times 4^3 =$ ______	$9^5 \times 9^6 =$ ______
10.	$2^9 \div 2^{-3} =$ ______	$3^8 \div 3^2 =$ ______	$12^4 \times 12^{10} =$ ______
11.	$5^4 \times 5^2 =$ ______	$10^7 \div 10^4 =$ ______	$11^3 \times 11^4 =$ ______
12.	$7^5 \div 7^2 =$ ______	$6^6 \times 6^3 =$ ______	$12^4 \div 12^2 =$ ______

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Lesson 1.1 Using Exponents

A **power** of a number represents repeated multiplication of the number by itself.

$6^4 = 6 \times 6 \times 6 \times 6$ and is read *6 to the fourth power.*

In exponential numbers, the **base** is the number that is multiplied, and the **exponent** represents the number of times the base is used as factor. In 6^4, 6 is the base and 4 is the exponent.

5^5 means 5 is used as a factor 5 times.

$5 \times 5 \times 5 \times 5 \times 5 = 3{,}125$ $5^5 = 3{,}125$

Write each power as a product of the factors.

	a	b	c
1.	3^3 ____________	5^5 ____________	6^1 ____________
2.	2^{12} ____________	3^8 ____________	3^6 ____________
3.	4^7 ____________	4^4 ____________	8^3 ____________

Use exponents to rewrite these expressions.

	a	b
4.	$24 \times 24 \times 24$ ________	$2 \times 2 \times 2 \times 2$ ________
5.	$3 \times 3 \times 3 \times 3 \times 3$ ________	5×5 ________
6.	$5 \times 5 \times 5 \times 5 \times 5 \times 5$ ________	$4 \times 4 \times 4$ ________

Find the value of each expression.

	a	b	c
7.	$8^3 =$ ____________	$9^4 =$ ____________	$10^2 =$ ____________

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NAME ______________________

Lesson 1.3 Negative Exponents

When a power includes a negative exponent, express the number as 1 divided by the base and change the exponent to positive.

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16} = 0.0625$$

To multiply or divide powers with the same base, combine bases, add or subtract the exponents, and then simplify.

$$2^{-3} \times 2^{-2} = 2^{-5} = \frac{1}{2^5} = 0.03125$$

$$2^{-4} \div 2^{-2} = 2^{-2} = \frac{1}{2^2} = 0.25$$

Rewrite each expression with a positive exponent. Then, solve. Round your answer to four decimal places.

	a	b	c
1.	3^{-2} = ______	6^{-3} = ______	8^{-2} = ______
2.	7^{-3} = ______	3^{-3} = ______	9^{-2} = ______
3.	4^{-3} = ______	5^{-2} = ______	2^{-3} = ______
4.	2^{-4} = ______	10^{-3} = ______	1^{-4} = ______

Find each product. Round your answer to five decimal places.

5.	$4^{-2} \times 4^{-3}$ = ______	$2^{-4} \times 2^{-1}$ = ______	$3^{-2} \times 3^{-3}$ = ______
6.	$6^{-2} \times 6^{-2}$ = ______	$5^{-2} \times 5^{-4}$ = ______	$3^{-2} \times 3^{-2}$ = ______
7.	$8^{-6} \times 8^{4}$ = ______	$7^{-5} \times 7^{2}$ = ______	$2^{-7} \times 2^{4}$ = ______

Find each quotient. Round your answer to five decimal places.

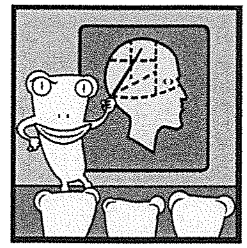
8.	$4^{-4} \div 4^{-2}$ = ______	$8^{-5} \div 8^{-3}$ = ______	$3^{-5} \div 3^{-2}$ = ______
9.	$2^{-8} \div 2^{-4}$ = ______	$5^{-6} \div 5^{-4}$ = ______	$6^{-7} \div 6^{-4}$ = ______
10.	$3^{-3} \div 3^{2}$ = ______	$4^{-3} \div 4^{1}$ = ______	$2^{-6} \div 2^{-3}$ = ______

EXPONENT RULES

GRAPHIC ORGANIZER

Name	Rule	Examples
ADDING & SUBTRACTING MONOMIALS	**COMBINE LIKE TERMS!!!** (DO NOT CHANGE common variables and exponents!)	**1.** $9x^2y - 10x^2y =$ **2.** Subtract 6w from 8w
PRODUCT RULE	$x^a \cdot x^b =$	**1.** $h^2 \cdot h^6 =$ **2.** $(-2a^2b) \cdot (7a^3b) =$
POWER RULE	$(x^a)^b =$	**1.** $(x^2)^3 =$ **2.** $(-2m^5)^2 \cdot m^3 =$
QUOTIENT RULE	$\frac{x^a}{x^b} =$	**1.** $\frac{27x^5}{42x} =$ **2.** $\frac{(y^2)^2}{y^4} =$
NEGATIVE EXPONENT RULE	$x^{-a} =$	**1.** $-5x^{-2} =$ **2.** $\frac{4k^2}{8k^5} =$
ZERO EXPONENT RULE	$x^0 =$	**1.** $7x^0 =$ **2.** $\frac{(w^4)^2}{w^8} =$

Alien Attack!



Aliens from the Outer Space Galactic Task Force have been watching the recent gains in mathematical understanding developing in human brains from Planet Earth. They have become alarmed that Planet Earth might soon develop the capabilities to discover that life exists on other planets with this increased mathematical knowledge. To slow the progress, they have organized an attack on the World Wide Web and all other forms of math textbooks. All words have been eradicated from the mathematics examples! Humans will now be forced to study the patterns of numbers and previously worked examples to rediscover the properties of mathematics! They are confident that humans will not persevere in the challenge of making sense of problems, reasoning abstractly and quantitatively, constructing viable arguments, looking for and making use of structure, and using repeated reasoning to recreate the language of mathematics. They have already declared MISSION ACCOMPLISHED!

The next standard in your mathematics class requires knowledge of the properties of integer exponents. Can your team recreate and name the properties by using the Standards for Mathematical Practice to examine the examples that remain?

Problem continues on next page. →

________________________ $7^3 \bullet 7^2 = (7 \bullet 7 \bullet 7) \bullet (7 \bullet 7) = 7^5$ $5^3 \bullet 5^3 = (5 \bullet 5 \bullet 5) \bullet (5 \bullet 5 \bullet 5) = 5^6$ $3 \bullet 3^4 = (3) \bullet (3 \bullet 3 \bullet 3 \bullet 3) = 3^5$	
________________________ $(2^2)^3 = (2^2) \bullet (2^2) \bullet (2^2) = 2^6$ $(6^4)^2 = (6^4) \cdot (6^4) = 6^8$	
________________________ $3^5 \bullet 2^5 = (3 \bullet 2)^5 = 6^5$ $4^3 \bullet 5^3 = (4 \bullet 5)^3 = 20^3$ $(2 \bullet 3)^6 = 2^6 \bullet 3^6$	

Problem continued from previous page.

Problem continues on next page. →

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Problem continued from previous page.

____	____	____
2^3	$2 \cdot 2 \cdot 2$	8
2^2	$2 \cdot 2$	4
2^1	2	2
2^0		
2^{-1}		
2^{-2}		
2^{-3}		
5^3	$5 \cdot 5 \cdot 5$	125
5^2	$5 \cdot 5$	25
5^1	5	5
5^0		
5^{-1}		
5^{-2}		
5^{-3}		

____

____

____

$$\frac{5^5}{5^2} = 5^{5-2} = 5^3 = 125$$

$$\frac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Lesson 2:

Scientific Notation

Step 1: Put the decimal after the first significant digit.

Step 2: Indicate how many places the decimal moved by the power of 10.

- A positive power of 10 indicates the decimal moved to the left.

$$300{,}000{,}000 \text{ m/s} = 3 \times 10^{8} \text{ m/s}$$

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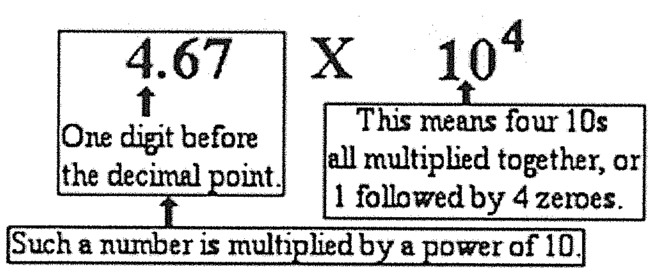
Numbers into Scientific Notation

0.0043 — The Number is a decimal less than 1, so the Exponent will be Negative.

$= 0.0043$ (3 places) — Move the Decimal point to the RIGHT to create a number between 1 and 10.

$= 0004.3$ — Remove Zeroes that are not needed.

$= 4.3 \times 10^{-3}$ ✓ — We moved 3 places so Power of 10 is three : 10^{-3}



Standard and Scientific Notation

Standard Form		Scientific Notation Form
1,600,000	=	1.6×10^6

1. **1,600,000** **Find where the original decimal point is**
   (If there isn't a decimal point, then it automatically goes to the far right of the number)
2. 1600000 Move the decimal point to the ***right of the first non-zero number***
3. **1.6×10^6** **Now rewrite the new number 1.6 times a power of 10**

- **The number of times you move decimal point (6 waves) will be the exponent value** ^{6}

Scientific Notation Form		Standard Form(Decimal)
3.54×10^{-9}	=	0.00000000354

1. **3.54×10^{-9}**
   - Notice the exponent number is how many places you will move the decimal
2. **0.000000003.54** Move decimal **left** if **exponent is Negative**
   Move decimal **right** if **exponent is Positive**
3. **0.00000000354** **Now the number is in Standard (Decimal) Form**

*Important to Note:

1. The number of places you must move the decimal point is the exponent you will use.
2. If number is fractional (less than 1 or greater than -1) than a negative exponent will be used.
3. If number is greater than 1 or smaller than -1, a positive exponent will be used.
4. If original number is positive keep it positive: $0.0012 = 1.2 \times 10^{-3}$
5. If original number is negative keep it negative: $-0.00039 = -3.9 \times 10^{-4}$

NAME ____________________

Lesson 1.4 Scientific Notation

Scientific notation is most often used as a concise way of writing very large and very small numbers. It is written as a number between 1 and 10 multiplied by a power of 10. Any number can be expressed in scientific notation.

$1{,}503 = 1.503 \times 10^3$ (+3) $0.0376 = 3.76 \times 10^{-2}$ (−2) $85 = 8.5 \times 10$ (+1)

Translate numbers written in scientific notation into standard form by reading the exponent.

$7.03 \times 10^5 = 703000$ Move the decimal right 5 places. $5.4 \times 10^{-4} = 0.00054$ Move the decimal left 4 places.

Write each number in scientific notation.

	a	b	c
1.	0.013 = ______	4105 = ______	27.3 = ______
2.	810.4 = ______	0.684 = ______	0.017 = ______
3.	0.0006 = ______	427.5 = ______	36,054 = ______
4.	50,210 = ______	0.0005 = ______	256.21 = ______
5.	36.25 = ______	0.892 = ______	0.00065 = ______
6.	0.027 = ______	1,416.3 = ______	0.0049 = ______

Write each number in standard form.

7.	2.6×10^{-3} = ______	8.46×10^5 = ______	4.65×10^{-1} = ______
8.	9.02×10^4 = ______	5.15×10^{-2} = ______	8.45×10^3 = ______
9.	7.25×10^{-4} = ______	1.06×10^3 = ______	9.06×10^{-5} = ______
10.	9.7×10^{-3} = ______	3.02×10^4 = ______	1.56×10^4 = ______

Star voyage — scientific notation

Name ____________________

Date ____________________

Problems

1. Write the following numbers in scientific notation.

Standard Notation	Scientific Notation
a. 93 000 000	____________
b. 384 000 000 000	____________
c. 0.00000000000234	____________
d. 0.0000000157	____________

2. Using the TI-30XS MultiView™ calculator, change the following numbers into scientific notation using SCI mode.

Standard Notation	Scientific Notation
a. 12 000 000	____________
b. 974 000 000	____________
c. 0.0000034	____________
d. 0.000000004	____________

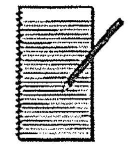
3. Using the TI-30XS MultiView calculator, change the following numbers into standard decimal notation using NORM mode.

Scientific Notation	Standard Notation
a. 5.8×10^{7}	____________
b. 7.32×10^{5}	____________
c. 6.2×10^{-6}	____________
d. 3×10^{-8}	____________

Star voyage — scientific notation

Name ______________________

Date ______________________



Problem

You are a captain of a starship in the distant future. You have been assigned to go to Alpha Centauri and you have 5 years to get there. The distance from our sun to Alpha Centauri is 2.5×10^{13} miles. The distance from the earth to our sun is approximately 9.3×10^{7} miles.

Although we have not yet discovered how to travel at the speed of light, you live in a time where your ship can travel at the speed of light.

Light travels the approximate distance of 6×10^{12} miles in 1 light year. You will take a path from earth by our sun and then on to Alpha Centauri. Will you be able to get to Alpha Centauri on time?

Procedure

1. Using the TI-30XS MultiView™ calculator, find the total distance that you need to travel. For this rough estimate, assume that you are measuring the distance as a straight line from the earth to our sun and then on to Alpha Centauri.

   ____

   Hint: Make sure your calculator is in scientific notation mode before you begin the calculation.

   Next, find out how long it will take you to travel the distance.
   (Distance traveled ÷ 1 light year)

   Hint: Make sure you use parentheses if needed in order to get the correct result for this division problem.

   ____

   ____

2. Can you make the trip in the allotted time of 5 years?

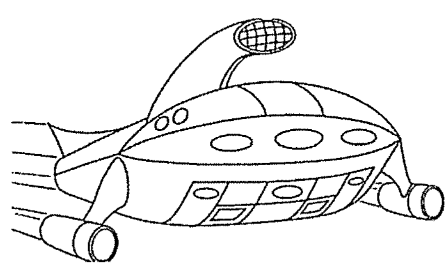
   ____

Extension

Now that you have been successful, you have been asked to make another trip. The distance from the Sun to Delta Centauri is 9×10^{13} miles. How long will it take you to get there from Earth?

Hint: The Earth is approximately 9.3×10^{7} miles from the Sun.

Your trip on this starship is fictitious. If you are interested in finding out more about the nearest star and cosmic distances, visit NASA web sites on the Internet.



Lesson 3:

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Difference between Rational and Irrational Number

Difference between Rational and Irrational Number

Rational numbers are numbers that can be expressed as a ratio of two integers. They can be in fraction, decimal or whole number from.

Example of rational number :- 1/2, 3/4 , -7/2, 8/1.

On the other side, irrational numbers are those numbers that cannot be expressed as a ratio of two integers.

Example of irrational number : - √2, π, 5/0. Apart from definition, there are some other differences also, which are given as:

Know More About What are Real Rational Numbers

TutorCircle

Difference between Rational and Irrational Number

Difference between Rational and Irrational Number

Rational numbers are numbers that can be expressed as a ratio of two integers. They can be in fraction, decimal or whole number from. Example of rational number:- 1/2, 3/4 , -7/2, 8/1.

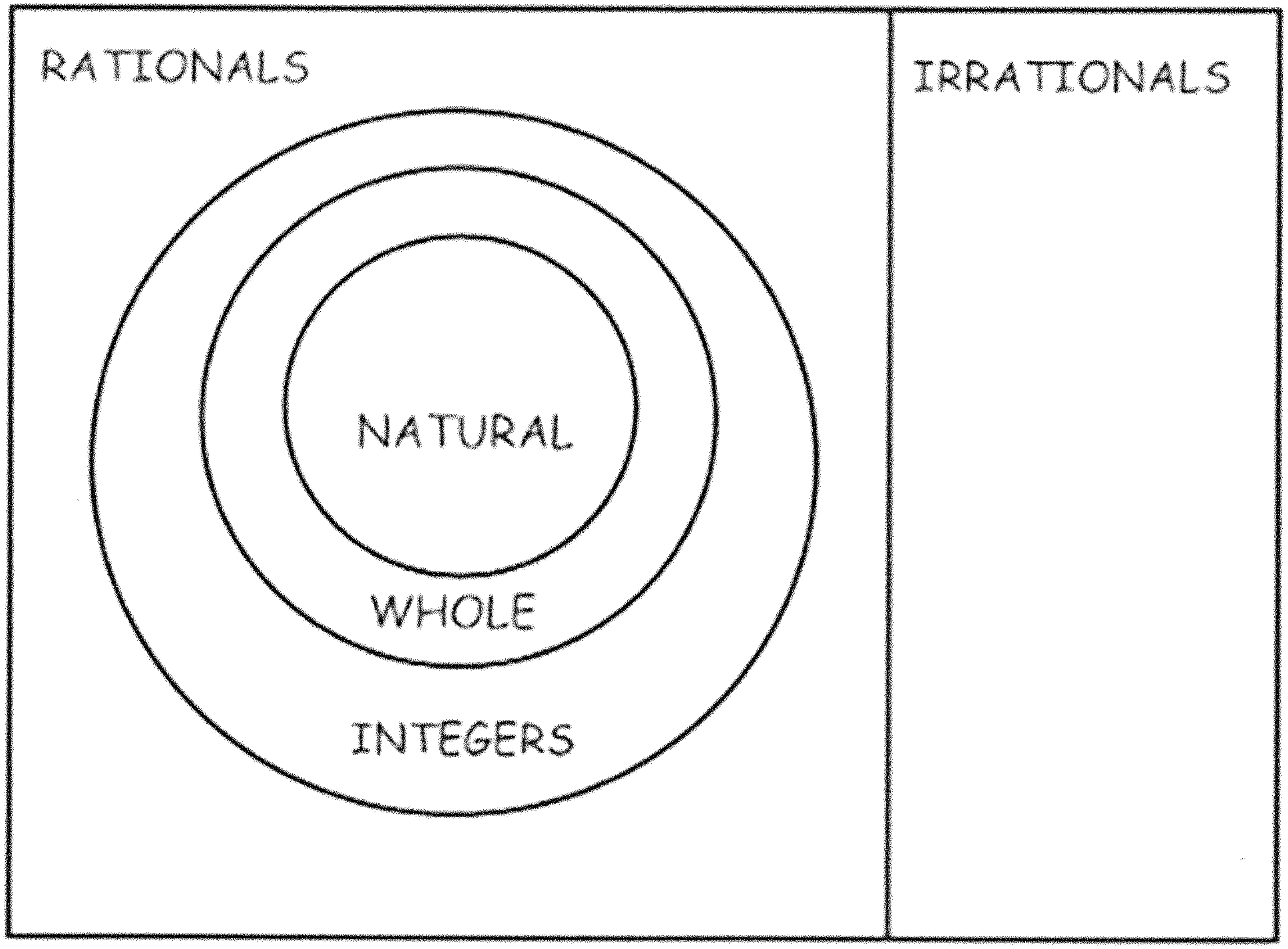
On the other side, irrational numbers are those numbers that cannot be expressed as a ratio of two integers. Example of irrational number: - √2, π, 5/0. Apart from definition, there are some other differences also, which are given as:

(1) Rational numbers are all positive and negative fraction, including integer and improper fraction.

An irrational number is a real number that cannot be written as a simple fraction.

Know More About Density Property of Real Number Worksheets

Real Number System



NAME ____________________

Lesson 2.1 Understanding Rational and Irrational Numbers

A **rational number** is a number that either terminates or repeats a pattern. It can be written as a fraction, $\frac{a}{b}$, where a and b are both whole number integers and b does not equal zero.

Here are some examples of rational numbers: $3, -5, \frac{1}{3}, 4.\overline{66}, \frac{5}{11}, 3.25$

An **irrational number** is any decimal that does not terminate and never repeats. These numbers are often represented by symbols.

Here are some examples of irrational numbers: $5.23143\ldots, \sqrt{5}, \pi$

Tell if each number is *rational* or *irrational.*

	a	b	c
1.	$\frac{1}{5}$ ______	$\sqrt{5}$ ______	-5 ______
2.	$\sqrt[3]{27}$ ______	$\frac{1}{3}$ ______	2.354 ______
3.	$\sqrt{36}$ ______	$3.\overline{45}$ ______	$\frac{7}{9}$ ______
4.	$\sqrt{20}$ ______	19.294153 ______	$-\frac{4}{5}$ ______
5.	$\sqrt{15}$ ______	π ______	$-\frac{7}{10}$ ______

NAME ______________________

Check What You Know

Rational and Irrational Numbers

Evaluate each expression.

	a	b	c
1.	$\sqrt{25} =$ ______	$\sqrt{9} =$ ______	$\sqrt{100} =$ ______
2.	$\sqrt{\frac{4}{16}} =$ ______	$\sqrt{81} =$ ______	$\sqrt{\frac{9}{25}} =$ ______

Approximate the value of each expression.

3. The value of $\sqrt{10}$ is between ______ and ______.

4. The value of $\sqrt[3]{74}$ is between ______ and ______.

Use roots or exponents to solve each equation. Write fractions in simplest form.

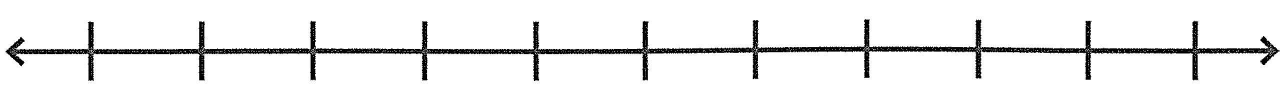
	a	b	c
5.	$x^2 = 64$	$\sqrt{x} = 9$	$x^3 = 343$
	x = ______	x = ______	x = ______

Compare using <, >, or =.

6.	$\sqrt{\frac{4}{9}}$ ______ $\frac{2}{3}$	$\sqrt{10}$ ______ 5	$\sqrt[3]{25}$ ______ 3

Put the values below in order from least to greatest along the number line.

7. 14, $\sqrt{18}$, 4π



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NAME ____________

Check What You Know

Rational and Irrational Numbers

CHAPTER 4 PRETEST

Solve each equation.

	a	b	c
1.	$9 + d = 16$ ________	$y + 3 = 9$ ________	$12 + a = 27$ ________
2.	$18 - b = 4$ ________	$23 - c = 21$ ________	$w - 11 = 11$ ________
3.	$n + 8 = 41$ ________	$7 + m = 20$ ________	$9 + s = 9$ ________
4.	$t + 18 = 5$ ________	$36 - a = 36$ ________	$15 - b = 0$ ________

Solve the following equations. Write each answer in simplest form.

5.	$m + 3.4 = 7.9$ ________	$n - 6.3 = 9$ ________	$p - (-\frac{6}{7}) = \frac{13}{14}$ ________
6.	$8t = \frac{1}{9}$ ________	$s \times \frac{3}{4} = -20$ ________	$-10.5r = -31.5$ ________

Solve each problem.

7. Ella has gold, silver, and copper wire for stringing beads. She has $1\frac{1}{2}$ ft. of gold wire, $2\frac{1}{3}$ ft. of silver wire, and $3\frac{3}{4}$ ft. of copper wire. How much wire does she have altogether?

She has ________ ft. of wire.

8. Green Valley Middle School wants to raise $7,500 for new equipment. If grades 6 and 7 each raise $2,450.25, how much money does grade 8 need to raise?

Grade 8 needs to raise ________.

NAME ______________________

Lesson 4.3 Understanding Rational and Irrational Numbers

A **rational number** is a number that either terminates or repeats a pattern. It can be written as a fraction, $\frac{a}{b}$, where *a* and *b* are both whole number integers and *b* does not equal zero.

Here are some examples of rational numbers: 3, -5, $\frac{1}{3}$, $4.\overline{66}$, $\frac{5}{11}$, 3.25

An **irrational number** is any decimal that does not terminate and never repeats. These numbers are often represented by symbols.

Here are some examples of irrational numbers: $5.231143...$, $\sqrt{5}$, π

Tell if each number is *rational* or *irrational.*

	a	b	c
1.	$\frac{1}{5}$ ________	$\sqrt{5}$ ________	-5 ________
2.	$\sqrt[3]{27}$ ________	$\frac{1}{3}$ ________	2.354 ________
3.	$\sqrt{36}$ ________	$3.\overline{45}$ ________	$\frac{7}{9}$ ________
4.	$\sqrt{20}$ ________	19.294153 ________	$-\frac{4}{5}$ ________
5.	$\sqrt{15}$ ________	π ________	$-\frac{7}{10}$ ________

NAME ______________________

Lesson 4.5 Comparing Rational and Irrational Numbers

Compare rational and irrational numbers by using a best guess for irrational numbers.

$\sqrt{3} < 2$ This statement is true because $\sqrt{3}$ is between 1 and 2.

$5 > \sqrt{20}$ This statement is true because $\sqrt{20}$ is between 4 and 5.

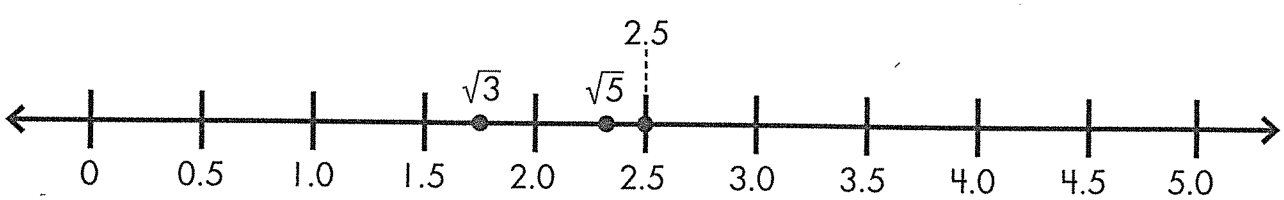
Approximate the value of an irrational number by exploring values.

The value of $\sqrt{2}$ is between 1 and 2. Look at the squares of 1.4 and 1.5.

$1.4^2 = 1.96$ $1.5^2 = 2.25$

By looking at these squares, it is evident that $\sqrt{2}$ is between 1.4 and 1.5.

Rational and irrational numbers can be compared by approximating their value and placing them along a number line, such as the numbers $\sqrt{5}$, 2.5, $\sqrt{3}$



Compare using <, >, or =.

	a	b	c
1.	$\sqrt{9}$ ________ π	4.5 ________ $\sqrt{25}$	3.9 ________ $\sqrt{10}$
2.	$\sqrt{2}$ ________ 1	$\sqrt[3]{\frac{8}{27}}$ ________ $\frac{2}{3}$	1.1 ________ $\sqrt{2}$

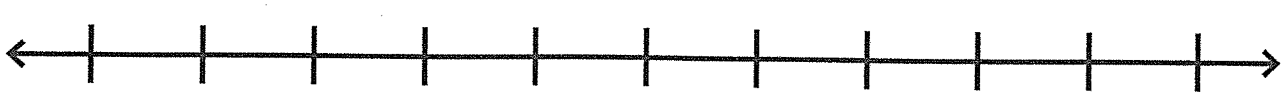
Approximate the value of each root to the tenths place.

3. The value of $\sqrt{7}$ is between ________ and ________.

4. The value of $\sqrt{10}$ is between ________ and ________.

Put the values below in order from least to greatest along a number line.

5. π^2, 10, $\sqrt{75}$



NAME ______________________

Check What You Learned

Rational and Irrational Numbers

Tell if each number is *rational* or *irrational*.

	a	b	c
1.	$\frac{7}{4}$ ________	π ________	$\sqrt{99}$ ________
2.	3.635 ________	$\sqrt{77}$ ________	$\sqrt{255}$ ________
3.	5.6 ________	$\frac{1}{9}$ ________	2.756 ________

Compare the values using <, >, or =.

	a	b	c
4.	$\sqrt{36}$ ______ 6.5	1.4 ______ $\sqrt{2}$	$\frac{1}{2}$ ______ 0.55
5.	3.9 ______ $\sqrt{10}$	$\sqrt{5}$ ______ 4	$\sqrt{8}$ ______ 3
6.	1 ______ $\frac{\sqrt{16}}{25}$	$\sqrt[3]{343}$ ______ 7.2	$\sqrt[3]{6}$ ______ 2

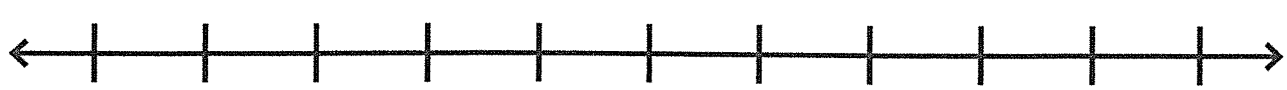
Approximate the value to the hundredths place.

7. The value of $\sqrt{5}$ is between ____________ and ____________.

8. The value of $\sqrt{13}$ is between ____________ and ____________.

Create a number line to show each set of values in order from least to greatest.

9. π, $\sqrt{10}$, -3, $\frac{7}{4}$



CHAPTER 4 POSTTEST

Lesson 4:

Chart of Perfect Squares 1 to 30

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$

Cube Roots 1 to 10

$\sqrt[3]{0} = 0$	$\sqrt[3]{64} = 4$	$\sqrt[3]{512} = 8$
$\sqrt[3]{1} = 1$	$\sqrt[3]{125} = 5$	$\sqrt[3]{729} = 9$
$\sqrt[3]{8} = 2$	$\sqrt[3]{216} = 6$	$\sqrt[3]{1000} = 10$
$\sqrt[3]{27} = 3$	$\sqrt[3]{343} = 7$	

Square and Square Root Table

Square	Square Root	Square	Square Root
$1^2 = 1$	$\sqrt{1} = 1$	$16^2 = 256$	$\sqrt{256} = 16$
$2^2 = 4$	$\sqrt{4} = 2$	$17^2 = 289$	$\sqrt{289} = 17$
$3^2 = 9$	$\sqrt{9} = 3$	$18^2 = 324$	$\sqrt{324} = 18$
$4^2 = 16$	$\sqrt{16} = 4$	$19^2 = 361$	$\sqrt{361} = 19$
$5^2 = 25$	$\sqrt{25} = 5$	$20^2 = 400$	$\sqrt{400} = 20$
$6^2 = 36$	$\sqrt{36} = 6$	$21^2 = 441$	$\sqrt{441} = 21$
$7^2 = 49$	$\sqrt{49} = 7$	$22^2 = 484$	$\sqrt{484} = 22$
$8^2 = 64$	$\sqrt{64} = 8$	$23^2 = 529$	$\sqrt{529} = 23$
$9^2 = 81$	$\sqrt{81} = 9$	$24^2 = 576$	$\sqrt{576} = 24$
$10^2 = 100$	$\sqrt{100} = 10$	$25^2 = 625$	$\sqrt{625} = 25$
$11^2 = 121$	$\sqrt{121} = 11$	$26^2 = 676$	$\sqrt{676} = 26$
$12^2 = 144$	$\sqrt{144} = 12$	$27^2 = 729$	$\sqrt{729} = 27$
$13^2 = 169$	$\sqrt{169} = 13$	$28^2 = 784$	$\sqrt{784} = 28$
$14^2 = 196$	$\sqrt{196} = 14$	$29^2 = 841$	$\sqrt{841} = 29$
$15^2 = 225$	$\sqrt{225} = 15$	$30^2 = 900$	$\sqrt{900} = 30$

LESSON 11.2

Approximating Square Roots

BEFORE

You found square roots of perfect squares.

Now

You'll approximate square roots of numbers.

WHY?

So you can find the falling speed of a skydiver, as in Ex. 22.

In the Real World

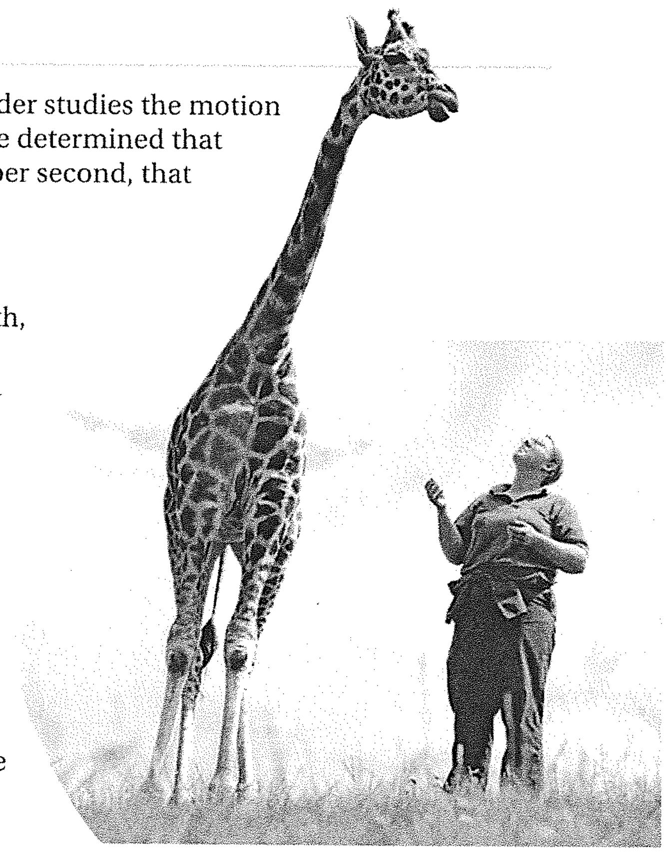
Word Watch

irrational number, p. 541
real number, p. 541

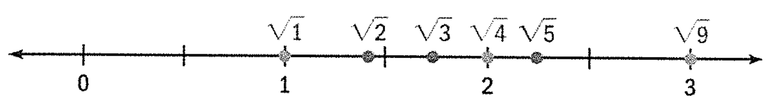
Animals Dr. R. McNeill Alexander studies the motion of animals. From his studies, he determined that the maximum speed s, in feet per second, that an animal can walk is

$$s = 5.66\sqrt{l}$$

where l is the animal's leg length, in feet. What is the maximum walking speed for a giraffe with a leg length of 11 feet? You'll find the answer in Example 3.



Evaluating Square Roots You know how to evaluate square roots like $\sqrt{1}$, $\sqrt{4}$, and $\sqrt{9}$ because 1, 4, and 9 are perfect squares. But what about square roots like $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$? The values of these square roots fall between whole numbers, as shown on the number line below.



EXAMPLE 1 Approximating to a Whole Number

Approximate $\sqrt{11}$ to the nearest whole number.

Make a list of whole numbers that are perfect squares: 0, 1, 4, 9, 16, ...

$9 < 11 < 16$	Identify perfect squares closest to 11.
$\sqrt{9} < \sqrt{11} < \sqrt{16}$	Take positive square root of each number.
$3 < \sqrt{11} < 4$	Evaluate square roots.

ANSWER Because 11 is closer to 9 than to 16, $\sqrt{11}$ is closer to $\sqrt{9} = 3$. So, to the nearest whole number, $\sqrt{11} \approx 3$.

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HELP with Solving

Once you find the approximation of a square root to the tenths' place, you can use the same method to find the approximation to the hundredths' place, thousandths' place, and so on.

EXAMPLE 2 Approximating to the Nearest Tenth

Approximate $\sqrt{11}$ to the nearest tenth.

You know from Example 1 that $\sqrt{11}$ is between 3 and 4. Make a list of squares of 3.1, 3.2, . . . , 3.9. From the list, you can see that 11 is between 3.3^2 and 3.4^2. So, $\sqrt{11}$ is between 3.3 and 3.4.

$3.1^2 =$	9.61
$3.2^2 =$	10.24
$3.3^2 =$	10.89
$3.4^2 =$	11.56
$3.5^2 =$	12.25

ANSWER Because 11 is closer to 10.89 than to 11.56, $\sqrt{11}$ is closer to $\sqrt{10.89} = 3.3$. So, to the nearest tenth, $\sqrt{11} \approx 3.3$.

Your turn now **Approximate the square root to the nearest whole number and then to the nearest tenth.**

1. $\sqrt{10}$ **2.** $\sqrt{22}$ **3.** $\sqrt{45}$ **4.** $\sqrt{115}$

EXAMPLE 3 Using Square Roots

You can use the approximation of $\sqrt{11}$ from Example 2 to estimate the maximum walking speed of the giraffe described on the previous page.

$s = 5.66\sqrt{l}$	Write maximum walking speed formula.
$= 5.66\sqrt{11}$	Substitute 11 for l.
$\approx 5.66(3.3)$	Use approximation of $\sqrt{11}$ to the nearest tenth.
≈ 19	Multiply.

ANSWER The maximum walking speed is about 19 feet per second.

Square Roots

Root	# of Squares	Root	# of Squares

VOCAB:
Square Root

Radical

Square Roots

Root	# of Squares	Root	# of Squares

VOCAB:
Square Root

Radical

NAME ______________________________

Lesson 4.4 Square and Cube Roots

The **square** of a number is that number times itself. A square is expressed as, for example, 6^2, which means 6×6, or 6 squared. The **square root** of a number is the value that, multiplied by itself, equals that number. The square root of 36 is 6. This is expressed as $\sqrt{36} = 6$.

Not all square roots of numbers are whole numbers, like 6. Numbers that have a whole number as their square root are called **perfect squares**.

The **cube root** of 27 is 3: $\sqrt[3]{27} = 3$, because $3 \times 3 \times 3 = 27$. Numbers that have a a whole number cube root are called **perfect cubes**.

The square root or cube root of any number that is not a perfect square is called a **radical number**. The symbols $\sqrt{\ }$ and $\sqrt[3]{\ }$ are called **radical signs**. When a number is not a perfect square, you can estimate its square root by determining which perfect squares it comes between. $\sqrt{50}$ is a little more than 7, because $\sqrt{49}$ is exactly 7. $\sqrt{60}$ is between 7 and 8 but is closer to 8, because 60 is closer to 64 than to 49. The same strategy can be used for cube roots.

A table of squares and square roots appears at the back of this book. Use the table to identify the square root of these perfect squares.

	a	b	c
1.	$\sqrt{9}$ = ________	$\sqrt{81}$ = ________	$\sqrt{36}$ = ________
2.	$\sqrt{25}$ = ________	$\sqrt{4}$ = ________	$\sqrt{64}$ = ________

Estimate the following square roots without looking at the table at the back of the book.

3. $\sqrt{80}$ is between ________ and ________ but closer to ________.

4. $\sqrt{27}$ is between ________ and ________ but closer to ________.

Identify the cube root.

5.	$\sqrt[3]{8,000}$ = ________	$\sqrt[3]{125}$ = ________	$\sqrt[3]{343}$ = ________
6.	$\sqrt[3]{8}$ = ________	$\sqrt[3]{64}$ = ________	$\sqrt[3]{1,000}$ = ________

NAME ___________________________

Lesson 2.2 Square Roots

The **square** of a number is that number times itself. A square is expressed as $\mathbf{6^2}$, which means 6×6, or 6 squared. The **square root** of a number is the number that, multiplied by itself, equals that number. The square root of 36 is 6: $\sqrt{36} = 6$.

Not all square roots of numbers are whole numbers like 6. Numbers that have a whole number as their square root are called **perfect squares**.

The expression of a square root is called a **radical**. The symbol $\sqrt{}$ is called a **radical sign**. When a number is not a perfect square, you can estimate its square root by determining which perfect squares it comes between.

$\sqrt{50}$ is a little more than 7, because $\sqrt{49}$ is exactly 7. $\sqrt{60}$ is between 7 and 8 but closer to 8, because 60 is closer to 64 than to 49.

Identify the square root of these perfect squares.

	a	b	c
1.	$\sqrt{16}$ = ______	$\sqrt{64}$ = ______	$\sqrt{25}$ = ______
2.	$\sqrt{100}$ = ______	$\sqrt{1}$ = ______	$\sqrt{9}$ = ______
3.	$\sqrt{36}$ = ______	$\sqrt{81}$ = ______	$\sqrt{4}$ = ______

Estimate the following square roots.

4. $\sqrt{85}$ is between ______ and ______ but closer to ______.
5. $\sqrt{20}$ is between ______ and ______ but closer to ______.
6. $\sqrt{35}$ is between ______ and ______ but closer to ______.
7. $\sqrt{70}$ is between ______ and ______ but closer to ______.
8. $\sqrt{45}$ is between ______ and ______ but closer to ______.

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NAME ______________________

Lesson 2.4 Using Roots to Solve Equations

Equations with exponential variables can be solved using the inverse operation. In this case, using roots will help to solve the problem.

$x^2 = 121$ **Step 1:** Evaluate the problem to find out which root to use. In this case, the exponent is 2, so you would use the square root as the inverse operation.

$\sqrt{x^2} = \sqrt{121}$ **Step 2:** Find the root of both sides of the equation.

$x = 11$ **Step 3:** Solve the problem.

Solve each problem by using roots. Show your work and write fractions in simplest form.

	a	b	c
1.	$x^2 = \frac{16}{169}$	$729 = x^3$	$x^2 = \frac{8}{125}$
	$x =$ ______	$x =$ ______	$x =$ ______
2.	$25 = x^2$	$x^2 = \frac{25}{64}$	$x^3 = 512$
	$x =$ ______	$x =$ ______	$x =$ ______
3.	$\frac{9}{36} = x^2$	$x^3 = 512$	$x^2 + 2 = 38$
	$x =$ ______	$x =$ ______	$x =$ ______
4.	$68 - 4 = x^3$	$x^2 - 5 = 44$	$x^3 + 4 = 5$
	$x =$ ______	$x =$ ______	$x =$ ______

NAME ______________________

Lesson 2.4 Using Roots to Solve Equations

Equations with exponential variables can be solved using the inverse operation. In this case, using exponents will help to solve the problem.

$\sqrt{x} = 6$ **Step 1:** Evaluate the problem to decide which exponent to use. In this case, since we are solving for the square root, the appropriate exponent to use will be 2 (or square).

$(\sqrt{x})^2 = 6^2$ **Step 2:** Square both sides of the equation.

$x = 36$ **Step 3:** Solve the problem.

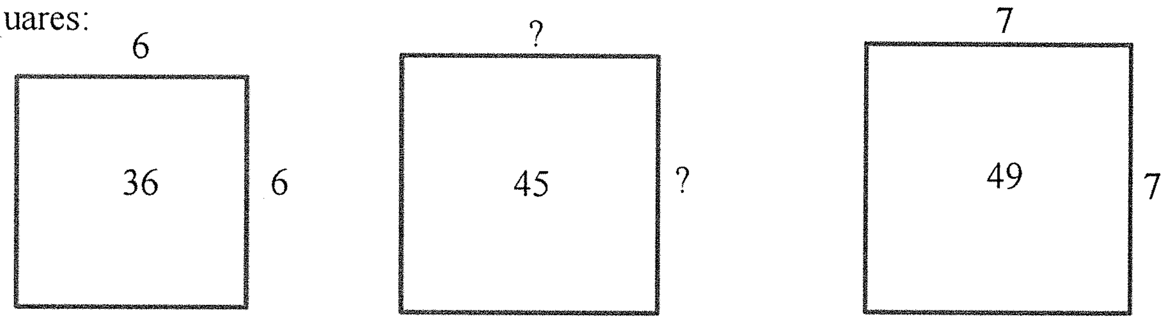
Solve each problem by using roots. Show your work and write fractions in simplest form.

	a	b	c
1.	$\sqrt{x} = 25$ $x =$ ________	$5 = \sqrt{x}$ $x =$ ________	$\sqrt[3]{x} = 6$ $x =$ ________
2.	$\sqrt{x - 4} = 4$ $x =$ ________	$\sqrt[3]{x} = 19$ $x =$ ________	$7 = \sqrt{x}$ $x =$ ________
3.	$\sqrt[3]{78 - x} = 4$ $x =$ ________	$18 = \sqrt{x}$ $x =$ ________	$6 = \sqrt{42 - x}$ $x =$ ________
4.	$8 = \sqrt[3]{x - 6}$ $x =$ ________	$\sqrt{x} = 14$ $x =$ ________	$7 = \sqrt[3]{x}$ $x =$ ________

* <u>Edges Of Squares And Cubes</u>

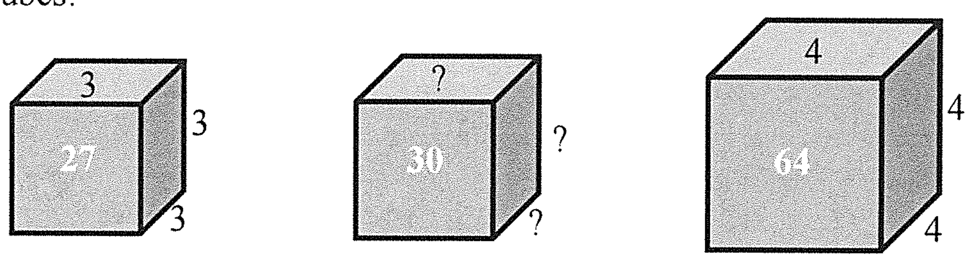
The area of each of the squares below is shown in the center of each square. The lengths of the sides of the first and last square are also given. Your task is to work with your partner, using a calculator, to find the length of the sides of the square in the center. You are not to use the square root key, but to estimate what you think the length of the side would be and test it by squaring it. Continue to estimate until you have found a value to the hundredths place that gets as close to 45 as possible.

Squares:



The volume of each of the cubes below is shown in the center of each cube. The lengths of the edges of the first and last cubes are also given. Your task is to work with your partner, using a calculator, to find the length of the edges of the cube in the center. You are not to use the cube root key, but to estimate what you think the length of the edge would be and test it by cubing it. Continue to estimate until you have found a value to the hundredths place that gets as close to 30 as possible.

Cubes:



Solutions will satisfy these equations:

$$\square \times \square = 45 \quad \text{or} \quad \square^2 = 45$$

and

$$\square \times \square \times \square = 30 \quad \text{or} \quad \square^3 = 30$$

Reflection:

1. How did you decide what the whole number part of your estimate should be for the square?

What do you think you could do to get a side length of the square that would produce a more accurate area?

Is it possible to find a side length that would be perfect for a square with an area of 45 square units? Explain your reasoning.

2. How did you decide what the whole number part of your estimate should be for the cube?

What do you think you could do to get an edge length for the cube that would produce a more accurate volume?

Is it possible to find an edge length that would be perfect for a cube with a volume of 30 cubic units? Explain your reasoning.