Unit 6:
Systems of Equations

2015-2016
8^{th} grade Mathematics
Worth Co. Middle School

OVERVIEW

In this unit students will:

- understand the solution to a system of equations is the point of intersection when the equations are graphed;
- understand the solution to a system of equations contains the values that satisfy both equations;
- find the solution to a system of equations algebraically;
- estimate the solution for a system of equations by graphing;
- understand that parallel lines have will have the same slope but never intersect; therefore, have no solution;
- understand the two lines that are co-linear share all of the same points; therefore, they have infinitely many solutions; and
- apply knowledge of systems of equations to real-world situations

STANDARDS FOR MATHEMATICAL CONTENT

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

BIG IDEAS

- There are situations that require two or more equations to be satisfied simultaneously.
- There are several methods for solving systems of equations.
- Solutions to systems can be interpreted algebraically, geometrically, and in terms of problem contexts.
- The number of solutions to a system of equations can vary from no solution to an infinite number of solutions.

ESSENTIAL QUESTIONS

- What does the point of intersection mean?
- What is a system of equations?
- What does it mean to solve a system of linear equations?
- How do I decide which method would be easier to use to solve a particular system of equations?
- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?
- How can I interpret the meaning of a "system of equations" algebraically and geometrically?
- What does the geometrical solution of a system mean?

3.1 Solving Systems Graphically

Now that we know how to solve complicated equations, we move on to solving what are called systems of equations. A system of equations is when we have multiple equations with multiple variables and we are looking for values that the variables represent so that all of the equations are true at the same time.

We will mainly be dealing with two variables and two equations, but you can solve most systems of equations as long as you have the same number of equations as variables. As a quick example, consider the following system:

$$x + y = 5$$

$$x - y = 1$$

It doesn't take too much work to verify the solution of this system is $x = 3$ and $y = 2$. Notice that those values for x and y make both equations true at the same time.

$$3 + 2 = 5$$

$$3 - 2 = 1$$

The question remains, how do we get that solution?

Solving with Graphs

If we have our equations set up using the x and y variables, we can graph both equations. Let's see how this helps us. To start with, let's graph the first equation $x + y = 5$. Remember that we can do this in a couple of ways. We could simply make an x/y chart and plot the points. Alternately, we could get the equation in slope-intercept form and then graph.

Let's start with an x/y chart. Remember that in an x/y chart we pick x values and substitute those into the equation to find y values. Confirm on your own that this x/y chart is correct for $x + y = 5$:

x	-2	-1	0	1	2
y	7	6	5	4	3

Now we can plot those points on a coordinate plane and connect them to get our graph.

If we don't like the x/y chart method, we can turn the equation into slope-intercept form by isolating the y variable on the left side like so:

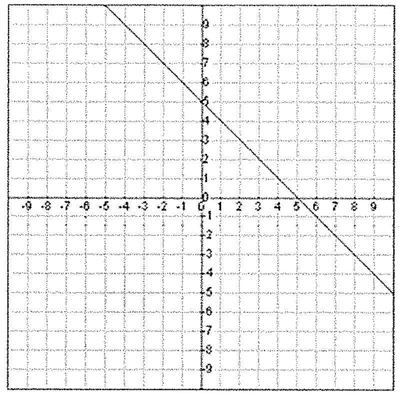
$$x + y = 5$$

$$x - x + y = 5 - x$$ ← Subtract x from both sides

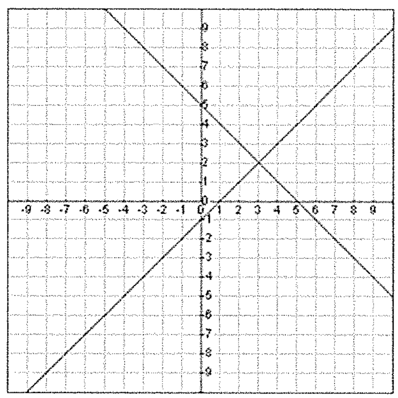
$$y = 5 + (-x)$$ ← Subtract means add a negative

$$y = -x + 5$$ ← Commutative property

Either way, we'll get a graph that looks like this:



Now we graph the second equation, $x - y = 1$, in the same way. It turns into $y = x - 1$ and gives us an overall graph like the following:



What do you notice about those two lines? They intersect. At what point do they intersect? The intersection is at the point (3,2) which means that $x = 3$ and $y = 2$. What does this tell us about solving systems of equations using graphs?

Yes, the point of intersection is the solution to the system because that point is the only point on both lines (assuming we're dealing with only linear equations for now). In fact, we sometimes write the solution to a system of equations as a point. So the solution to this system is (3,2).

Let's try another example. What is the solution to the following system of equations?

$$4x + 2y = 6$$

$$-x + 2y = -4$$

We'll leave it as an exercise to verify that the following equations are the same system just written in slope-intercept form:

$$y = -2x + 3$$

$$y = \frac{1}{2}x - 2$$

Now graph those equations to see where they intersect.

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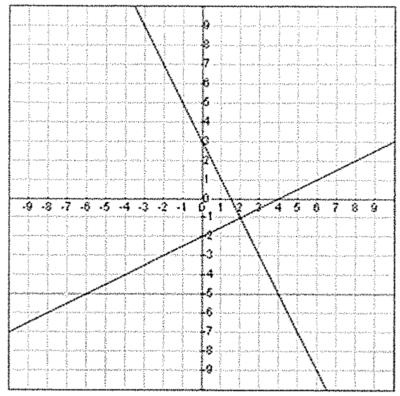


It looks like the graphs intersect at the point (2,-1) which we can verify by substituting into the original equations as follows:

$$4x + 2y = 6 \rightarrow 4(2) + 2(-1) = 6$$

$$-x + 2y = -4 \rightarrow -(2) + 2(-1) = -4$$

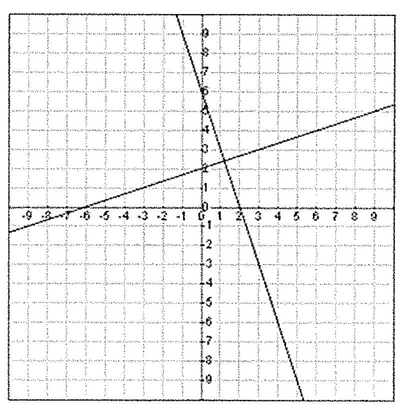
That means we have the correct solution.

Estimating Using a Graph

So far our solutions have been integer values, but that won't always be the case. We can still use the graphing method to get a decent estimate even if it's not a very nice solution. For example, consider the following equations and graphs.

$$y = \frac{1}{3}x + 2$$

$$y = -3x + 6$$



Note that the x coordinate where the lines intersect is a little more than 1 and the y coordinate of intersection is little more than 2. We might estimate this solution as $(1\frac{1}{4}, 2\frac{1}{3})$ or the decimal equivalent. The actual solution is $(1.2, 2.4)$ for this system, but we'll discover how to find the exact solution later.

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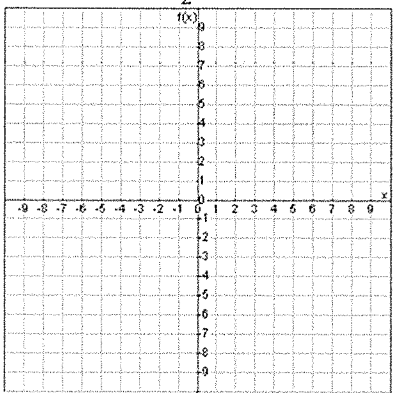
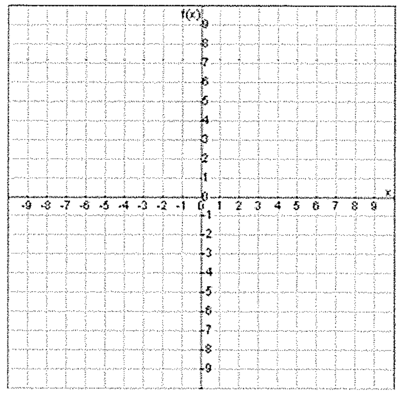
Lesson 3.1

Graph the following systems of equations and estimate the solution from the graph.

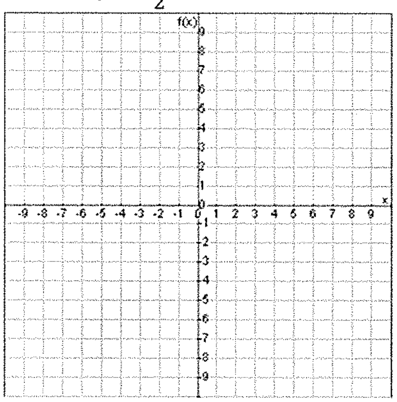
1. $y = x + 1$
   $y = -\frac{3}{2}x + 6$



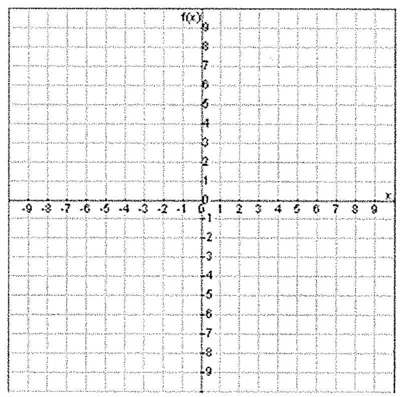
2. $y = 2x + 8$
   $y = x + 6$



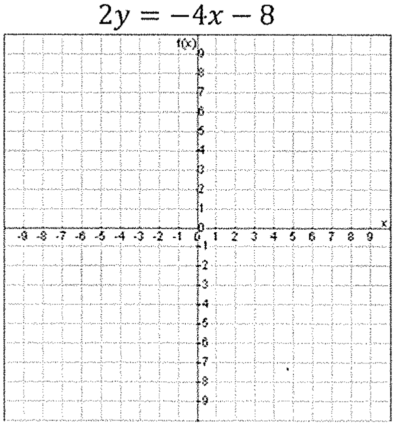
3. $y = -2x + 3$
   $y = \frac{1}{2}x - 4$



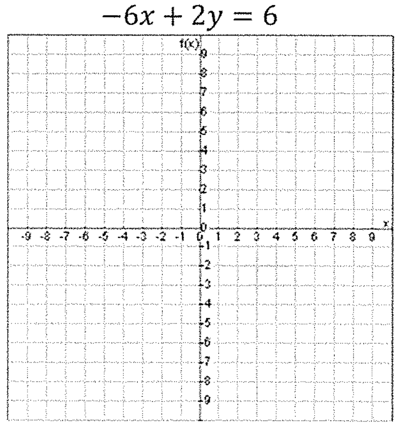
4. $y = 2x + 3$
   $y = 4$



5. $3y = x + 9$
   $2y = -4x - 8$



6. $4x + 2y = 6$
   $-6x + 2y = 6$



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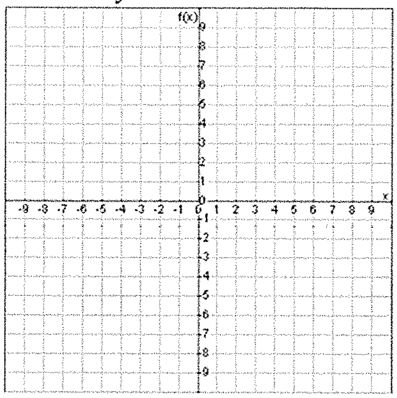
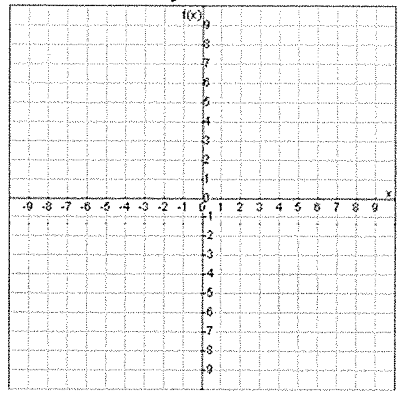
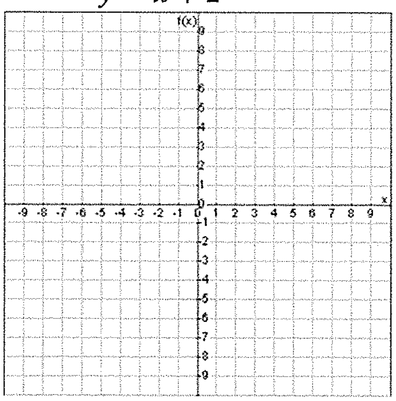
7. $3y = 6$
$2y = -x + 12$



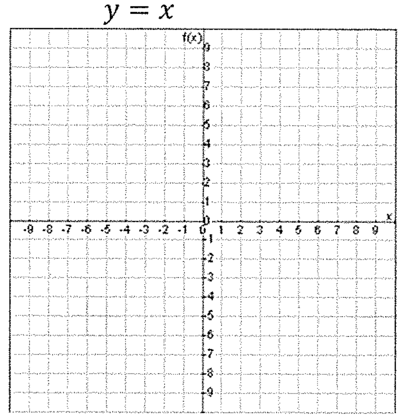
8. $-x + 3y = -18$
$x + 2y = -4$



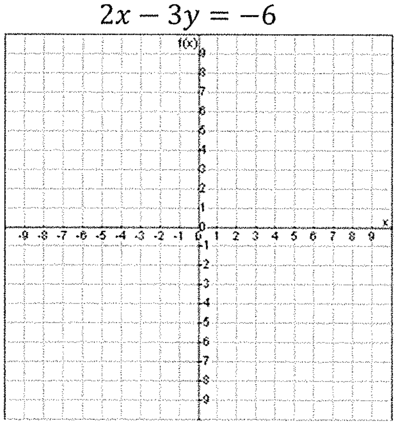
9. $y = -x$
$y = x + 2$



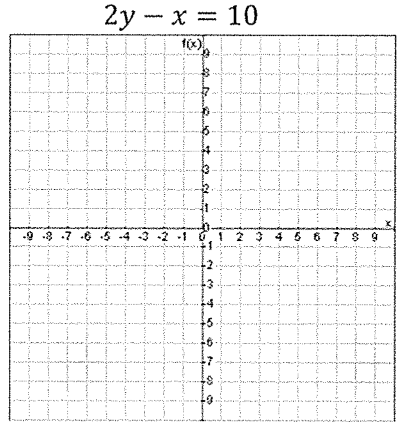
10. $x + y = 6$
$y = x$



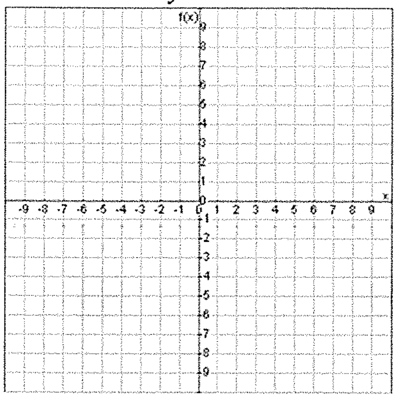
11. $4x + 3y = 24$
$2x - 3y = -6$



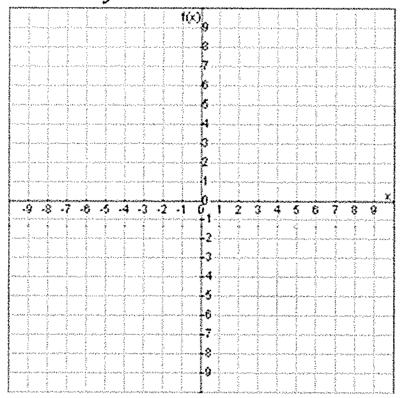
12. $x + y = 2$
$2y - x = 10$



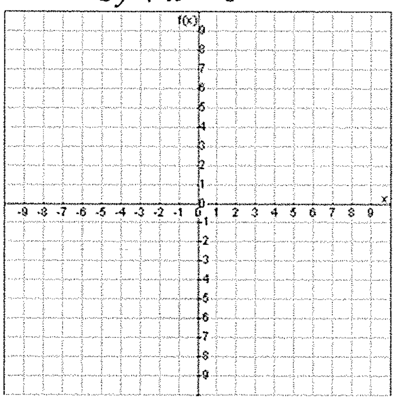
13. $x - 2y = 2$
$3x + y = 6$



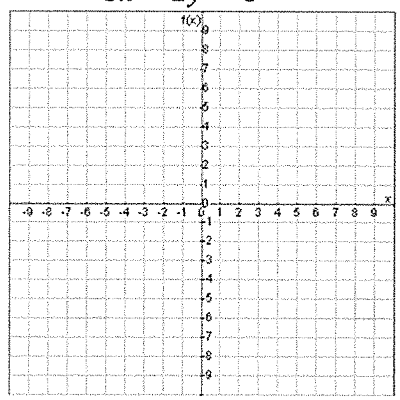
14. $y = -4$
$y = -3x - 4$



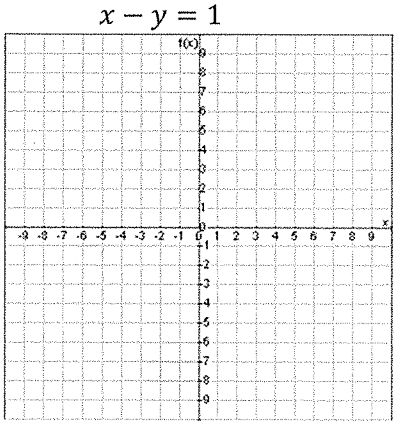
15. $y = x + 3$
$3y + x = 6$



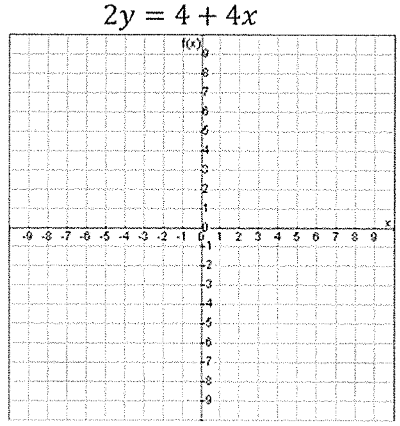
16. $x + 5y = -5$
$3x - 2y = 8$



17. $-x + y = -1$
$x - y = 1$



18. $4x = 2y - 10$
$2y = 4 + 4x$



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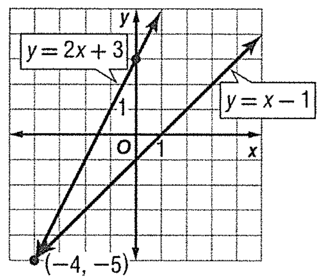
Reteach

Solve Systems of Equations by Graphing

Example

Solve the system $y = 2x + 3$ and $y = x - 1$ by graphing.

Graph each equation on the same coordinate plane.



The graphs appear to intersect at (−4, −5).

Check this estimate by replacing x with −4 and y with −5.

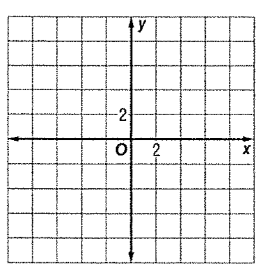
Check	$y = 2x + 3$	$y = x - 1$
	$-5 \stackrel{?}{=} 2(-4) + 3$	$-5 \stackrel{?}{=} -4 - 1$
	$-5 = -5$ ✓	$-5 = -5$ ✓

The solution of the system is (−4, −5).

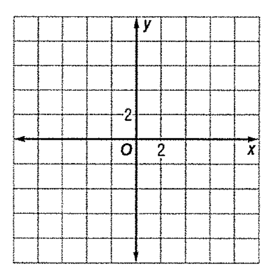
Exercises

Solve each system of equations by graphing.

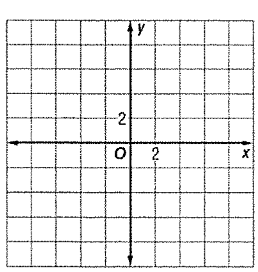
1. $y = 2x + 5$
$y = -x + 8$



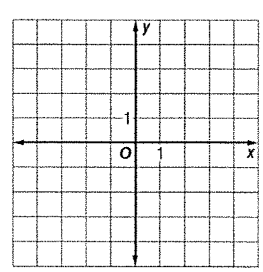
2. $y = -x - 3$
$y = x + 1$



3. $y = -3x + 9$
$y = -3x + 3$



4. $y = -2x + 4$
$y = -x + 3$



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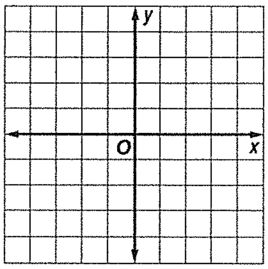
Skills Practice

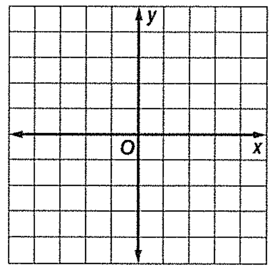
Solve Systems of Equations by Graphing

Solve each system of equations by graphing.

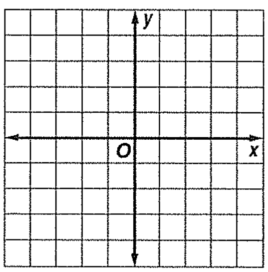
1. $y = x + 4$
$y = -2x - 2$



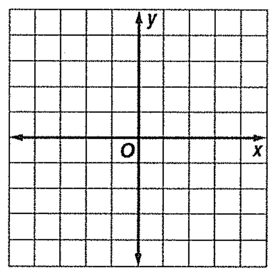
2. $y = 5x - 1$
$y = 5x + 10$



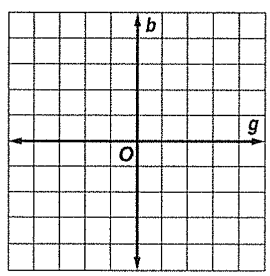
3. $y = x - 1$
$y - x = -1$



4. $y = 6x - 3$
$y = -3$



5. **CLUBS** There are thirty-three students in the Chess Club. There are five more boys than girls in the club. Write and solve a system of equations to find the number of boys and girls in the Chess Club.



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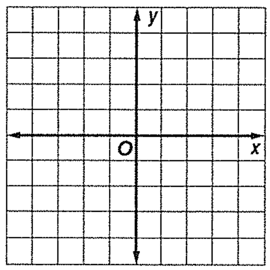
Homework Practice

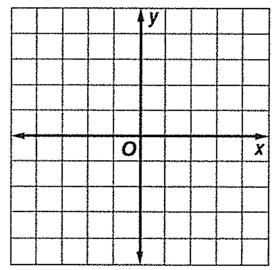
Solve Systems of Equations by Graphing

Solve each system of equations by graphing.

1. $y = 3x + 4$
$y = -x - 4$

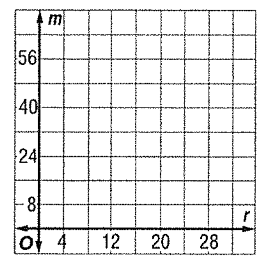


2. $y = 10 + 6x$
$y = 6x$

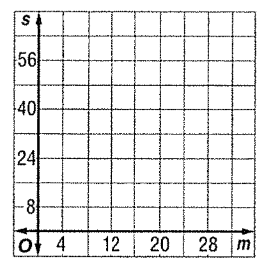


Write and solve a system of equations that represents each situation. Interpret the solution.

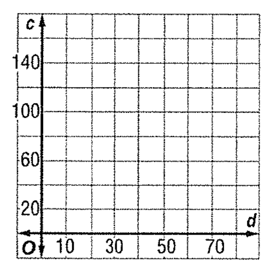
3. **BASKETBALL** Alonzo and Miguel scored a total of 54 points in the basketball game. Miguel scored four more points than Alonzo.



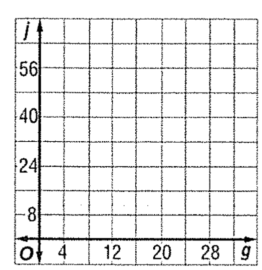
4. **AGES** Morgan is 15 years younger than Mrs. Santos. Their combined age is 44.



5. **ANIMALS** The total number of cats and dogs at the shelter is 125. There are 5 more cats than dogs.



6. **PING-PONG** Jenny won the ping-pong championship eight more times than Gerardo. They have won a combined total of 32 championships.



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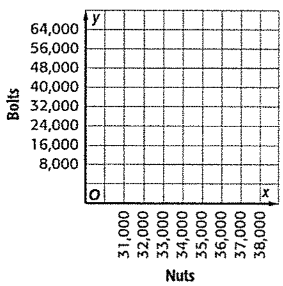
Problem-Solving Practice

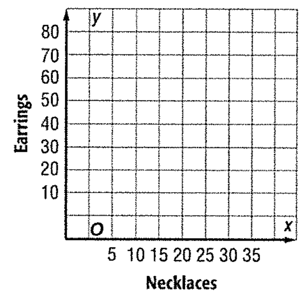
Solve Systems of Equations by Graphing

Write and solve a system of equations that represents each situation. Interpret the solution.

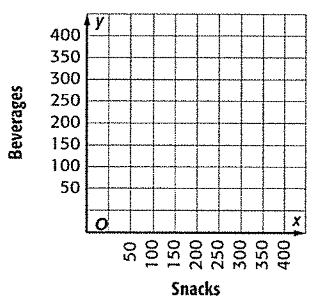
1. PROFIT Mr. Blackwell's company produces nuts and bolts. The total monthly profit for his company was $76,378. The profit earned from nuts was $3,428 more than the profit earned from bolts.



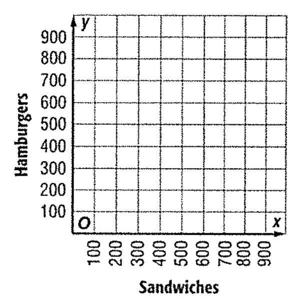
2. JEWELRY Julie has 81 pieces of jewelry. She has twice as many earrings as she has necklaces.



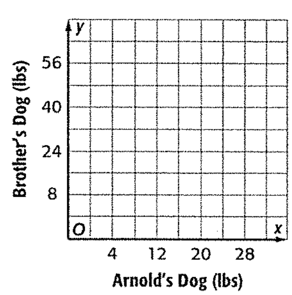
3. REFRESHMENTS The seventh grade class supplied bags of snacks and beverages for the school dance. They supplied 19 more beverages than bags of snacks. The dance was supplied with a total of 371 items.



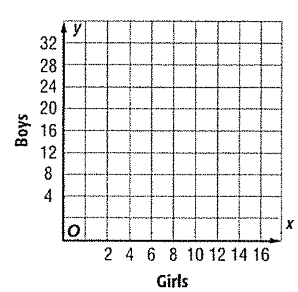
4. SANDWICHES The hamburger shop sells 500 sandwiches each day. They sell 100 more hamburgers than they do chicken sandwiches.



5. DOGS Arnold's dog weighs 10 pounds less than twice his brother's dog. The dogs' combined weight is 50 pounds.



6. STUDENTS There are 26 students in Mrs. Ortlieb's class. There are two more boys than girls.



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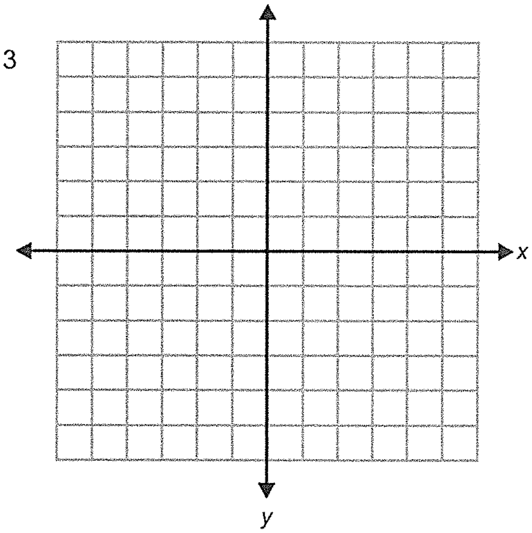
Systems Of Linear Equations

Directions: Change each equation into slope-intecept form.
Graph each system of linear equations to find the solution to each system.

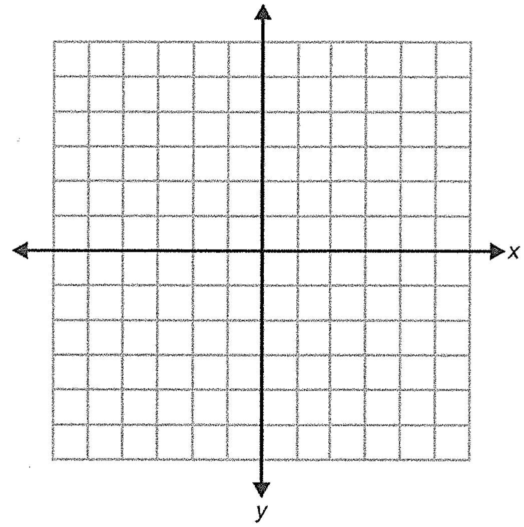
1) $-3x + y = 6x + 3$

$-8x + 4y = 16$



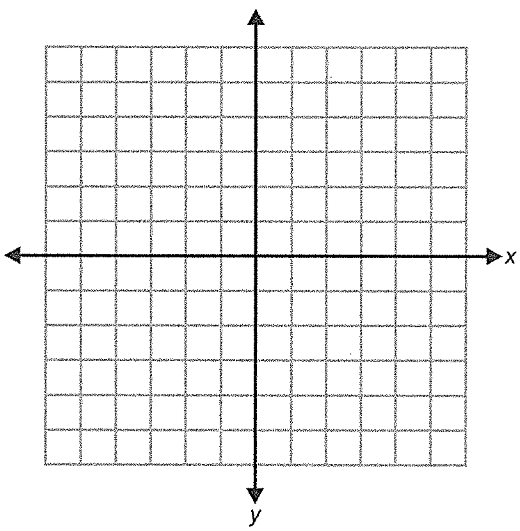
2) $5x + 2y = 10$

$y - x = -2$



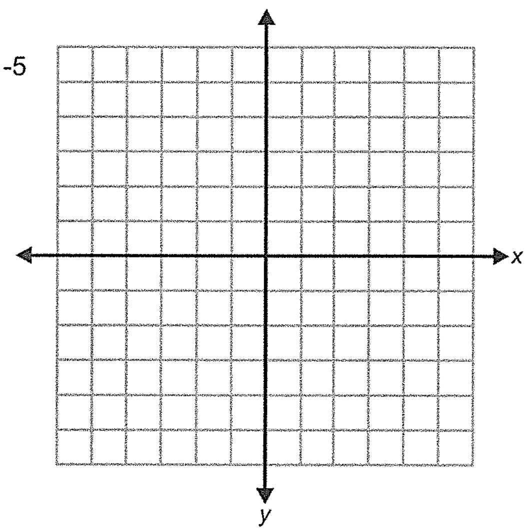
3) $-4x - 2y = -8$

$y + 5 = x$



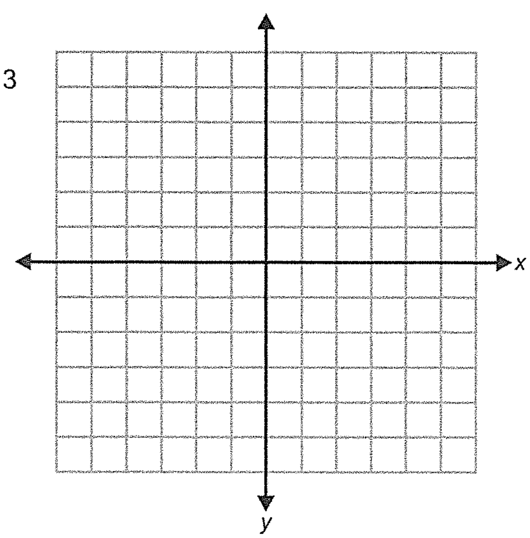
4) $3x + 9y + 4 = -5$

$3y + 2x = -9$



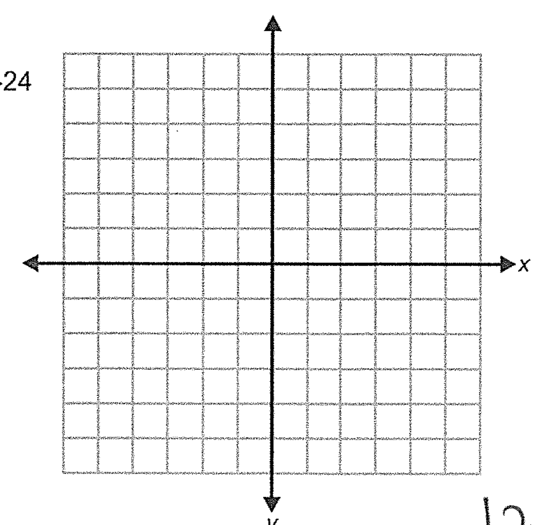
5) $-6y + 2x + 3 = 3$

$2y - 4x = 10$



6) $9y - 3x + 3 = -24$

$3y - 12 = 8x$



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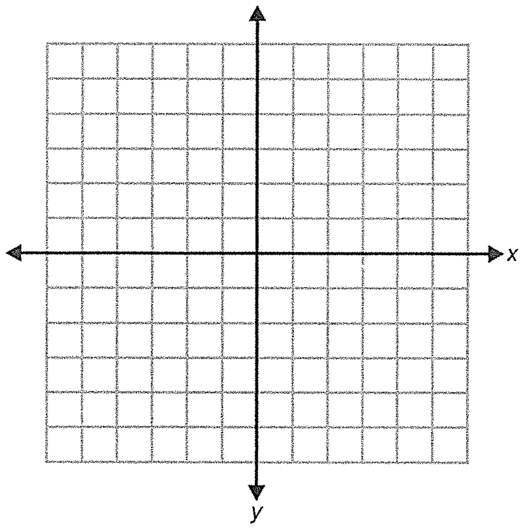
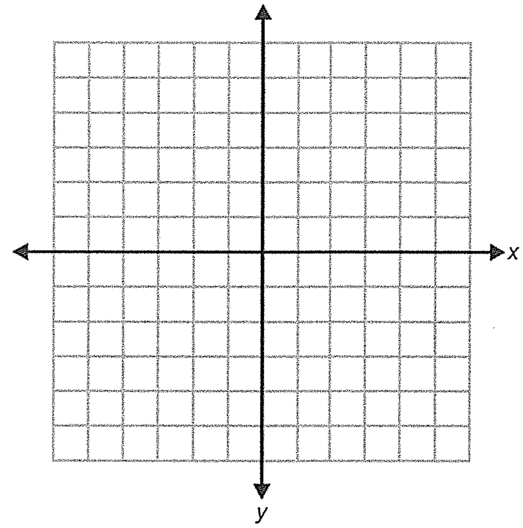
7) $-3x + 2y = 8$

$y = -5$



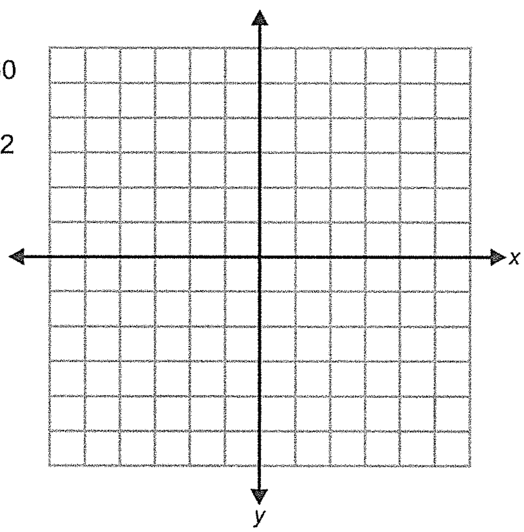
8) $3y + 2x = 9$

$3y + 9 = 4x$



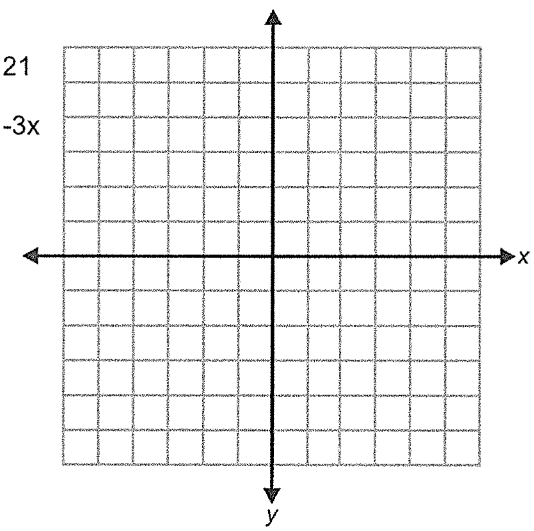
9) $20y + 10x = 60$

$4y - 8x = 12$



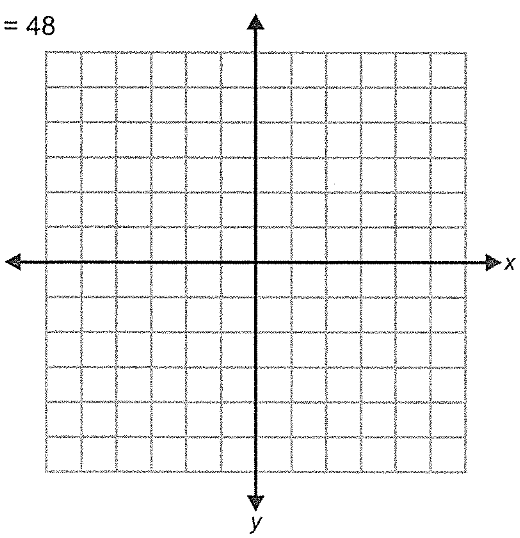
10) $-14x + 7y = 21$

$2y + 8 = -3x$



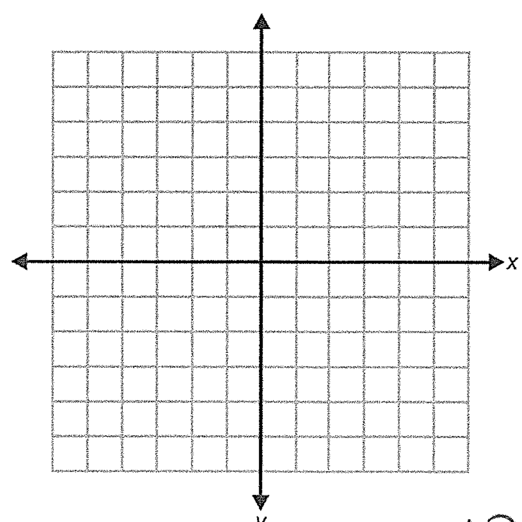
11) $100x + 50y - 2 = 48$

$y + x = 3$



12) $y + \frac{2}{5}x = 4$

$5y = 2x$



3.2 Solving Systems with Substitution

While graphing is useful for an estimate, the main way that we can solve a system to get an exact answer is algebraically. There are a few useful methods to do this, and we will begin with the substitution method. The general idea with this method is to isolate a single variable in one equation and substitute that into the other equation.

Isolating a Variable

Consider the following system of equations.

$$3x + y = 1$$

$$3x + 2y = 4$$

It is always best to check if one variable has a coefficient of one and isolate that variable. Remember that a coefficient is a number multiplied by a variable. That means that a coefficient of one will mean that the variable doesn't have a number in front of it because the one is understood to be there, and we don't write it. In this case, notice that the y in the first equation has a coefficient of one. It would probably be easiest to isolate that variable. Let's do so.

$$3x + y - 3x = 1 - 3x$$

$$y = 1 - 3x$$

Substitution

Now that we know what y is equal to in the first equation, we can substitute that expression for y in the second equation. Be careful to not plug back into the first equation or else we'll end up with infinite solutions every time. Since we want a solution that is true in both equations, we must use both equations.

$$3x + 2y = 4$$

$$3x + 2(1 - 3x) = 4$$

$$3x + 2 - 6x = 4$$

$$-3x + 2 = 4$$

Now that we have it down to a simple two-step equation, we can solve like normal and get the following:

$$-3x + 2 - 2 = 4 - 2$$

$$\frac{-3x}{-3} = \frac{2}{-3}$$

$$x = -\frac{2}{3}$$

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Finding the Second Variable Value

Now that we know what x equals, we can substitute that back into either of the original equations to find what the y coordinate is at the point of intersection of the two lines. It is also a good idea to plug in this x value into both equations to make sure they give the same y value. We'll start with the first equation.

$$3x + y = 1$$

$$3\left(-\frac{2}{3}\right) + y = 1$$

$$-2 + y = 1$$

$$-2 + 2 + y = 1 + 2$$

$$y = 3$$

Double Check

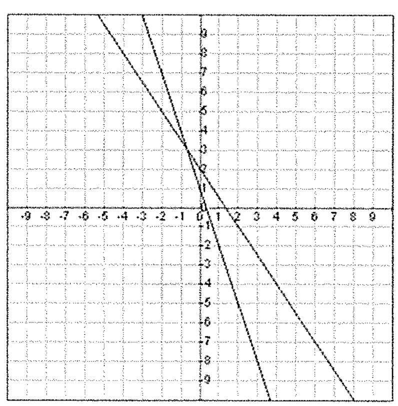
This means that the solution should be the point $(-\frac{2}{3}, 3)$, but we found that y value using the first equation. We need to make sure this point is on the second line as well, so let's substitute the values into that equation.

$$3x + 2y = 4$$

$$3\left(-\frac{2}{3}\right) + 2(3) = 4$$

$$-2 + 6 = 4$$

That statement is true and therefore the point is on the second line as well. So our solution is the point $(-\frac{2}{3}, 3)$ for this system. Just for some extra confidence, examine the following graph of the system of equations and notice that the point we found is indeed the point of intersection.



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Coefficients Other Than One

It may be the case that we have all coefficients with values other than one. We can still use substitution, but we'll have to be a bit more careful isolating one variable at the beginning. Let's consider the following system of equations.

$$2x + 4y = 8$$

$$3x + 2y = 7$$

In this case, it might be easier to solve the first equation for x because the coefficient for y and the 8 will easily divide by 2. So let's isolate the x in the first equation as follows:

$$2x + 4y - 4y = 8 - 4y$$

$$\frac{2x}{2} = \frac{8 - 4y}{2}$$

$$x = 4 - 2y$$

Now substitute that x value into the second equation as follows:

$$3x + 2y = 7$$

$$3(4 - 2y) + 2y = 7$$

$$12 - 6y + 2y = 7$$

$$12 - 4y = 7$$

$$12 - 12 - 4y = 7 - 12$$

$$-4y = -5$$

$$\frac{-4y}{-4} = \frac{-5}{-4}$$

$$y = \frac{5}{4}$$

16.

Now that we have the y coordinate, we can plug in to find the x value.

$$2x + 4y = 8$$

$$2x + 4\left(\frac{5}{4}\right) = 8$$

$$2x + 5 = 8$$

$$2x + 5 - 5 = 8 - 5$$

$$2x = 3$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$x = \frac{3}{2}$$

So our solution is $\left(\frac{3}{2}, \frac{5}{4}\right)$. We'll leave it as an exercise to double check using the second equation.

Infinite and No Solutions

It is still possible to get infinite solutions or no solution for a system of equations. After the substitution step, if we get down to a number equals a number statement that is always true, there are infinite solutions. If we get down to a number equals a number statement that is false, there are no solutions. This is really the application of what we learned earlier in this unit about solving equations with one variable and getting infinite or no solutions.

Lesson 3.2

Solve the following systems using the substitution method.

1. $2x + 8y = 12$
   $x - 2y = 0$

2. $x + y = 7$
   $2x + y = 5$

3. $y = 5$
   $2x - y = 9$

4. $y = -\frac{1}{2}x + 1$
   $2x + 3y = 6$

5. $2x + y = -16$
   $x - 2y = -28$

6. $4y = 8$
   $2x + 5y = 11$

7. $x + y = 2$
   $-2x + 4y = -19$

8. $x + 2y = 4$
   $3x - 4y = -3$

9. $2x + y = 4$
   $2y = -4x + 8$

10. $x + y = 2$
    $x + y = 5$

11. $y = 3x$
    $3x + 3y = 4$

12. $y = 2x + 3$
    $y = 4x - 1$

13. $x - 3y = 0$
    $\frac{1}{3}x + y = 2$

14. $2x - \frac{1}{3}y = -9$
    $-3x + y = 15$

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15. $x = 2$
$2x + y = 4$

16. $4x = 3y + 3$
$x = 2$

17. $\frac{3}{2}x = 2y$
$y = x - 1$

18. $x - 2y = -1$
$3y = x + 4$

19. $x + 2y = 0$
$3x + 4y = 4$

20. $2y = -6$
$x + 2y = -1$

21. $x - 4y = 1$
$2x - 8y = 2$

22. $x - 2y = 3$
$4x - 8y = 12$

23. $x = 0$
$3x - 6y = 12$

24. $x = 2y - 3$
$x = 2y + 4$

25. $2x - 3y = -24$
$x + \frac{1}{4}y = -5$

26. $\frac{2}{3}x - 2y = 12$
$x = -2y - 2$

27. $x + y = 6$
$2y = -2x + 2$

28. $x + 2y = 7$
$2x - 8y = 8$

29. $2x = 6y - 14$
$3y - x = 7$

30. $y = -x + 3$
$2y + 2x = 4$

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Write and solve a system of equations using any method (graphing, elimination, or substitution) for each of the following situations.

31. Leonard sells small watermelons for $7 each and large watermelons for $10 each. One day the number of small watermelons he sold was fifteen more than the number of large watermelons, and he made a total of $394. How many small and how many large watermelons did he sell?

32. The perimeter of a rectangle is 28 cm. The length of the rectangle is 2 cm more than twice the width. Find the dimensions of the rectangle.

33. The sum of Julian's and Kira's age is 58. Kira is fourteen less than twice as old as Julian. What are their ages?

34. A 3% solution of sulfuric acid was mixed with an 18% solution of sulfuric acid to produce an 8% solution. How much 3% solution and how much 18% solution were used to produce 15 L of 8% solution?

35. Supplementary angles are two angles whose measures have the sum of 180 degrees. Angles X and Y are supplementary, and the measure of angle X is 24 degrees greater than the measure of angle Y. Find the measures of angles X and Y.

36. At the end of the 2000 baseball season, the New York Yankees and the Cincinnati Reds had won a total of 31 World Series. The Yankees had won 5.2 times as many World Series as the Reds. How many World Series did each team win?

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37. Peanuts worth $2.25 a pound were mixed with cashews worth $3.25 a pound to produce a mixture worth $2.65 a pound. How many pounds of each kind of nuts were used to produce 35 pounds of the mixture?

38. Ernesto spent a total of $64 for a pair of jeans and a shirt. The jeans cost $6 more than the shirt. What was the cost of the jeans?

39. The perimeter of a rectangular garden is 68 feet. The length of the garden is 4 more than twice the width. What are the dimensions of the garden?

40. The Future Teachers of America Club at Paint Branch High School is making a healthy trail mix to sell to students during lunch. The mix will have three times the number of pounds of raisins as sunflower seeds. Sunflower seeds cost $4.00 per pound, and raisins cost $1.50 per pound. If the group has $34.00 to spend on the raisins and sunflower seeds, how many pounds of each should they buy?

Name________________________________ Date __________

Solving Systems Using Substitution

Example A

Solve the following system of equations.

$$y = 3x - 10$$
$$y = 2x - 5$$

When both equations start with *y* by itself or isolated, simply take the part after the equal signs for each equation and set them equal to each other.

$$3x - 10 = 2x - 5$$

Next, isolate the x-variable to find the value of x.

$$3x - 10 = 2x - 5$$
$$-2x \qquad -2x$$
$$x - 10 = -5$$
$$+10 \quad +10$$
$$x = 5$$

Now that we know that x is equal to 5 for each equation, plug 5 into either of the two original equations and solve for y.

$y = 3x - 10$ ← one of the original equations.

$$y = 3(5) - 10$$
$$y = 15 - 10$$
$$y = 5$$

Now that we know the value for x and y for the given system of equations, we must state our answer.

$$x = 5 \quad y = 5$$

Our answer may also be stated as an ordered pair.

(5,5)

Example B

Solve the following system of equations.

$$y = x - 7$$
$$2x + y = 8$$

When one equation is in y = form and the other is in standard form and *y* is not isolated, you may use substitution to solve.

Because the first equation states that y is equal to x - 7, we can substitute x - 7 in place of the y variable in the equation 2x + y = 8.

$y = x - 7$

y is the same thing as x -7.

Therefore, we can replace the y from the other equation with x -7.

Now we only have one variable making it possible solve!

$$2x + y = 8$$
$$2x + (x - 7) = 8$$
$$2x + x - 7 = 8$$
$$3x - 7 = 8$$
$$+7 \quad +7$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

Now that we know that x is equal to 5 for each equation, plug 5 into either of the two original equations and solve for y.

$y = x - 7$ ← one of the original equations.

$y = (5) - 7$ ← substitute 5 for x.

$y = -2$

Now that we know the value for x and y for the given system of equations, we must state our answer.

$$x = 5 \quad y = -2$$

Our answer may also be stated as an ordered pair.

(5,-2)

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Directions: Solve each system of equations by using substitution.

1) $y = 4x + 11$
$-6x + 8y = 36$

2) $y = 5x + 8$
$y = x - 8$

3) $y = -2x + 5$
$y = -x + 3$

4) $y = 3x - 4$
$y = -2x + 1$

5) $y = 2x$
$y = x - 1$

6) $y = x - 5$
$3y + x = 1$

7) $y = -5x + 1$
$8x + 2y = -2$

8) $y = 4x - 7$
$y = 2x + 9$

9) $y = 3x$
$y = x + 2$

10) $y = x - 7$
$2x + y = 8$

11) $y = 2x + 5$
$6x - 4y = -18$

12) $y = x - 3$
$y = -3x + 25$

13) $y = x$
$x + y = 4$

14) $y = -x + 7$
$y = 2x - 8$

15) $y = 6x - 3$
$y = 3x + 6$

16) $y = -2x + 3$
$y = x + 12$

17) $y = 9x - 7$
$8x + 2y = 64$

18) $y = -4x - 3$
$4x - 2y = -18$

19) $y = 7x + 8$
$y = 4x + 5$

20) $y = -3x + 8$
$y = 2x - 7$

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Reteach

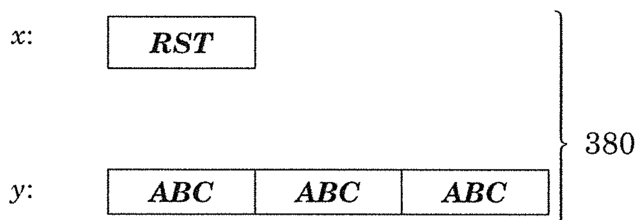
Solve Systems of Equations Algebraically

Example

You own three times as many shares of *ABC* stock as you do of *RST* stock. Altogether you have 380 shares of stock.

a. Write a system of equations to represent this situation.

Draw a bar diagram.



Use the diagram to write the system.

$y = 3x$ — There are 3 times as many shares *ABC* stocks as *RST* stocks.

$x + y = 380$ — The total number of stocks owned is 380.

b. Solve the system algebraically. Interpret the solution.

Since y is equal to $3x$, you can replace y with $3x$ in the second equation.

$x + y = 380$	Write the equation.
$x + 3x = 380$	Replace y with $3x$.
$4x = 380$	Simplify.
$\frac{4x}{4} = \frac{380}{4}$	Division Property of Equality
$x = 95$	Simplify.

Since $x = 95$ and $y = 3x$, then $y = 285$ when $x = 95$. The solution of this system of equations is (95, 285). This means that you own 95 shares of *RST* stock and 285 shares of *ABC* stock.

Exercises

Solve each system of equations algebraically.

1. $y = x + 3$
$y = 4x$

2. $y = -x - 2$
$y = -2x$

3. $y = x + 14$
$y = 8x$

4. $y = x - 6$
$y = 2x$

5. $y = -x + 8$
$y = 3x$

6. $y = -x$
$y = -2x$

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Skills Practice

Solve Systems of Equations Algebraically

Solve each system of equations algebraically.

1. $y = x - 8$
$y = 5x$

2. $y = -x - 4$
$y = 3x$

3. $y = x + 11$
$y = 12x$

4. $y = x - 14$
$y = -6x$

5. $y = -x + 9$
$y = 2x$

6. $y = x + 15$
$y = -4x$

7. $y = -x - 10$
$y = 4x$

8. $y = x + 24$
$y = -7x$

9. $y = -x + 18$
$y = 8x$

Write and solve a system of equations that represents each situation. Interpret the solution.

10. **TELEVISION** Videl watched 6 times as many hours of television over the weekend as Dineen. Together they watched a total of 14 hours of television. How many hours of television did each person watch over the weekend?

11. **CROSS-COUNTRY SKIING** Lucida is a cross-country ski racer. On Saturday, she skied twice as many miles as she did on Sunday. Over the weekend she skied a total of 63 miles. How far did she ski on each day?

12. **DARTS** Bryson and Lilly played a game of darts, and Lilly scored 4 more points than Bryson. The total of their scores was 180. How many points did each of them score?

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Homework Practice

Solve Systems of Equations Algebraically

Solve each system of equations algebraically.

1. $y = x + 2$
$y = -3x$

2. $y = -x$
$y = -7x$

3. $y = -x - 4$
$y = x$

4. $y = x - 6$
$y = 2x$

5. $y = x + 5$
$y = -2x$

6. $y = x - 4$
$y = 2x$

7. $y = -x - 14$
$y = -8x$

8. $y = x + 20$
$y = 6x$

9. $y = -x - 3$
$y = 3x$

Write and solve a system of equations that represents each situation. Interpret the solution.

10. MONEY Neil has a total of twelve \$5 and \$10 bills in his wallet. He has 5 times as many \$10 bills as \$5 dollar bills. How many of each does he have?

11. HAYRIDE Hillary and 23 of her friends went on a hayride. There are 8 more boys than girls on the ride. How many boys and girls were on the ride?

12. DRIVING Winston drove a total of 248 miles on Monday. He drove 70 fewer miles in the morning than he did in the afternoon. How many miles did he drive in the afternoon?

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Problem-Solving Practice

Solve Systems of Equations Algebraically

1. **GEOMETRY** The perimeter of a rectangle is 36 meters. The length of the rectangle is 4 meters longer than the width. Find the length and width of the rectangle.

2. **WOOD** Mildred cut a 9 foot board into two pieces. The long piece is twice as long as the short one. How long is the short piece?

3. **SWIMMING POOLS** Victor's swimming pool holds 3,000 gallons. He filled the pool using two hoses. The larger hose filled the pool four times as fast as the smaller one. How many gallons of water came from the smaller hose?

4. **FALL** Julio bought a total of 20 medium and large pumpkins. If he spent $53 and bought 6 more large pumpkins as medium pumpkins, how many large pumpkins did he buy?

Pumpkins
Large - $3
Medium - $2
Small - $1

5. **MUSIC** Mr. Winkle downloaded 34 more songs than Mrs. Winkle downloaded. Together they downloaded 220 songs. How many songs did each download?

6. **BAND** The seventh and eighth grade bands held a joint concert. Together there were 188 band members. If the eighth grade band is 3 times as big as the seventh grade band, how big is the eighth grade band?

7. **WORK** Amal worked a total of 30 hours last week. On Saturday and Sunday he worked 5 times as many hours than he worked the rest of the week. How many hours did he work the rest of the week?

8. **RAIN** During the months of August and September the total rainfall was 6.2 inches. If the rainfall in August was 0.6 inch more than the amount of rainfall in September, how much rain fell in each month?

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3.3 Solving Systems with Elimination

Sometimes it is easier to eliminate a variable entirely from a system of equations rather than use the substitution method. We do this by adding opposite coefficients together to get zero of one variable.

Subtracting to Eliminate

We first need to decide which variable is easiest to eliminate. Consider the following system of equations.

$$3x + y = 1$$

$$3x + 2y = 4$$

Notice that in this case the coefficients for x are the same. This means that they will be easily eliminated. If we subtract $3x$ from both sides of the first equation, we will eliminate the variable from the left side but will still have x are the right side. However, if we instead subtracted $(3x + 2y)$ from the left side we could subtract 4 from the right side because we know that $3x + 2y$ is exactly equal to 4 thanks to the second equation. (Remember that if we're going to solve a system of equations, we'll have to use both equations somehow, which is what we just did.)

This is sort of like repossession in a way. If you don't have the money to pay the bank, they can repossess your property up to an equivalent value of what you owe. In the same way, if we don't want to take away $(3x + 2y)$ from the right side, we can take away something equivalent which is 4 in this case. Let's take a look.

$$\begin{aligned} 3x + y &= \quad 1 \\ -(3x + 2y) &\quad -4 \\ 0x - 1y &= -3 \end{aligned}$$

Notice that it almost looks like we subtracted the second equation from the first. What we actually did was subtract expressions that are equal from both sides to keep the first equation balanced. Now we can solve since we have zero x's left.

$$-1y = -3$$

$$\frac{-1y}{-1} = \frac{-3}{-1}$$

$$y = 3$$

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Now that we know what y equals, we can substitute that back into either equation to find the x value of the solution point.

$$3x + y = 1$$

$$3x + (3) = 1$$

$$3x + 3 - 3 = 1 - 3$$

$$3x = -2$$

$$\frac{3x}{3} = \frac{-2}{3}$$

$$x = -\frac{2}{3}$$

So we get the solution $\left(-\frac{2}{3}, 3\right)$ which can verify is in the second equation by substituting both values in to make sure it is a true mathematical statement.

Adding to Eliminate

Adding to eliminate a variable will work the same way. In this case we should find one variable with the opposite coefficient of the same variable in the other equation. For example, consider this system of equations:

$$3x + 2y = 4$$

$$x - 2y = 4$$

Notice that the first equation has 2 as the coefficient for y and the second equation has a -2 as the coefficient. That means we should be able to add $(x - 2y)$ to both sides of the first equation. However, remember that we don't want to end up with more of the y variable on the right side, so we will add something equivalent to it. In this case that will be 4.

$$3x + 2y = \quad 4$$

$$+(x - 2y) \quad + 4$$

$$4x = 8$$

We get $0y$ which means that the y variable is eliminated

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

We'll leave it as an exercise to show that from here we can get $y = -1$ which means that our solution to this system of equations is the point $(2, -1)$.

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When the Coefficients Don't Match

The elimination method works fine when the coefficients match or are opposites, but what about when it is just a messy system of equations like this?

$$2x + 3y = -1$$

$$4x + 5y = -1$$

Solving this system by the substitution method would mean dealing with fractions and the coefficients don't match so it looks like the elimination method won't work either. However, is there a way we can get the coefficients to match?

Notice that in the first equation we have a $2x$ and in the second we have a $4x$. Wouldn't it be nice if the first equation had a $4x$ instead of the $2x$? Is there any way we can make that happen? If we multiply both sides of the first equation by 2, we will maintain equality and have $4x$ to match the second equation. Let's do so.

$$2(2x + 3y) = 2(-1)$$

$$4x + 6y = -2$$

Now that we have matching coefficients we can use the elimination method to continue to solve by subtracting $(4x + 6y)$ from the left side of our new equation and subtracting -2 from the right side since that is equal to $4x + 6y$.

$$4x + 6y = \quad -2$$

$$-(4x + 5y) \quad -(-1)$$

$$0x + 1y = -1$$

$$y = -1$$

From here we can again substitute $y = -1$ into either original equation to find that $x = 1$ which gives us the solution of $(1, -1)$.

Infinite and No Solutions

It is still possible to get infinite solutions or no solution for a system of equations. If both variables get eliminated and we get down to a number equals a number statement that is always true, there are infinite solutions. If we get down to a number equals a number statement that is false, there are no solutions.

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Lesson 3.3

Solve the following systems using the elimination method.

1. $x + y = 1$
   $x - y = 5$

2. $2x + 3y = 7$
   $-2x + y = 5$

3. $3x + y = 6$
   $3x - 2y = 9$

4. $\frac{1}{2}x + 3y = 1$
   $3x + 3y = 6$

5. $x + y = -3$
   $x - y = 1$

6. $4x + y = -9$
   $4x + 2y = -10$

7. $\frac{1}{5}x + 2y = -10$
   $2x + 2y = -10$

8. $-2x + y = 10$
   $4x + y = -8$

9. $-4x = 4$
   $4x - 3y = -10$

10. $x = 1$
    $6x - 5y = 11$

11. $x - 2y = 5$
    $3x - 2y = 9$

12. $3x + y = 5$
    $2x + y = 10$

13. $x = 5$
    $2x - 3y = 16$

14. $3x + \frac{3}{2}y = 6$
    $3x - 2y = -1$

15. $4x - 3y = 12$
$\frac{2}{3}x + 2y = 12$

16. $-5x + 3y = 6$
$x - y = 4$

17. $3y = 6$
$4x - y = -2$

18. $3x + y = 2$
$6x + 3y = 5$

19. $x + y = 4$
$2x + 2y = 8$

20. $x + y = 2$
$2x + 2y = 8$

21. $x + 3y = 12$
$2x - 3y = 12$

22. $2x + 3y = 10$
$5x + 7y = 24$

23. $5x + 4y = -3$
$10x - 2y = -3$

24. $5x - 4y = -8$
$3x + 8y = 3$

25. $4x - 7y = 10$
$3x + 2y = -7$

26. $\frac{1}{2}x - 3y = -4$
$4y = 8$

27. $3x - 4y = -10$
$5x + 8y = -2$

28. $4x + 3y = 19$
$3x - 4y = 8$

29. $4x + \frac{3}{2}y = 17$
$6x + 5y = 20$

30. $3x + 4y = -25$
$2x = -6$

Write and solve a system of equations using any method (graphing, elimination, or substitution) for each of the following situations.

31. The sum of two numbers is 82 and their difference is 26. Find each of the numbers.

32. Kathryn buys 8 cups of coffee and 2 bagels one day and pays $31. Harry buys 3 cups of coffee and 3 bagels the same day and pays $17.25. How much is each cup of coffee and each bagel?

33. Farmer Deanna looks out her window and counts a total of 64 legs on a total of 20 animals. If she has only sheep and chickens, how many of each does she have? *(Hint: Sheep have 4 legs each and chickens 2 legs each.)*

34. Tyler and Pearl went on a 20-kilometer bike ride that lasted 3 hours. Because there were so many steep hills on the bike ride, they had to walk for most of the trip. Their walking speed was 4 kilometers per hour. Their riding speed was 12 kilometers per hour. How much time did they spend walking?

35. A used book store also started selling used CDs and videos. In the first week, the store sold 40 used CDs and videos at $4.00 per CD and $6.00 per video. The sales for both CDs and videos totaled $180.00. How many CDs and videos did the store sell in the first week?

36. A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper?

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37. Dried apricots worth $3.25 a pound were mixed with dried prunes worth $4.75 a pound to produce a mixture of dried fruit worth $3.79 a pound. How much of each kind of fruit was used to produce 25 pounds of mixture?

38. One number added to twice another number is 23. Four times the first number added to twice the other number is 38. What are the numbers?

39. The owners of the River View Restaurant have hired enough servers to handle 17 tables of customers, and the fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat and how many four-seat tables should the owners purchase?

40. The Rodriguez family and the Wong family went to a brunch buffet. The restaurant charges one price for adults and another price for children. The Rodriguez family has two adults and three children, and their bill was $40.50. The Wong family has three adults and one child, and their bill was $38.00. Determine the price of the buffet for an adult and the price for a child.

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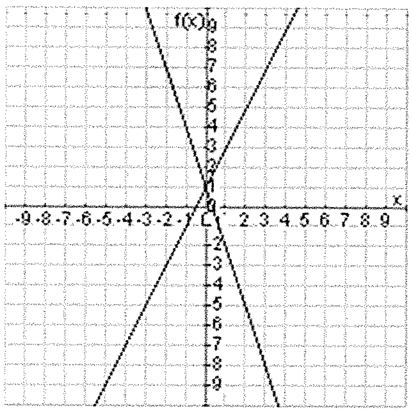
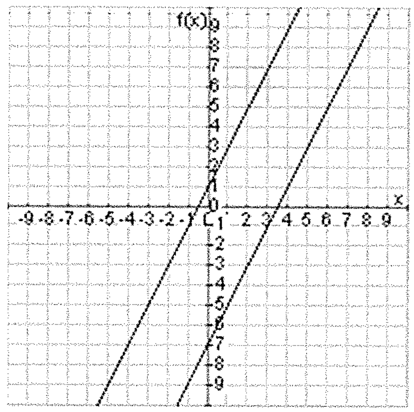
3.4 Solving Systems by Inspection

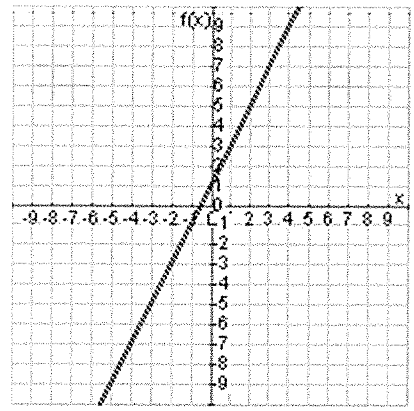
What makes a system of linear equations have a single solution, no solutions, or infinite solutions? One of the first representations we looked at for systems was the graphical representation. What is true about the following systems of linear equations that have either infinite or no solution?



One solution
$y = 2x + 1$
$y = -3x + 1$



No solutions
$y = 2x + 1$
$y = 2x - 7$



Infinite solutions
$y = 2x + 1$
$y = 2x + 1$

Notice that the systems with no solutions and infinite solutions both have the same slope. In other words, the lines are parallel. If those parallel lines have different y-intercepts, then there are no solutions to the system. If those parallel lines are in fact the same exact line (same slope and same y-intercept), then there are infinite solutions. The lines are sitting right on top of each other. Therefore if we could quickly determine whether two lines have the same slope, we could know if it will have infinite or no solutions.

Standard Form

If the two equations are given in slope-intercept form, then we can readily see the slope and y-intercept. Same slope and different intercept would mean no solution. Same slope and same intercept would mean infinite solutions. However, not all equations are given in slope-intercept form. Another common form of a linear equation is called **standard form**, which is: $Ax + By = C$.

Consider the following system of equations given in standard form. We can't readily see the slope or y-intercept since they are both in standard form.

$$2x + y = 5$$

$$4x + 2y = 10$$

So how can we find the slope? We could solve each equation for y, but this method is called inspection. We're looking for a quicker way. Let's get the second equation in slope-intercept form and see if we can find any patterns of where the slope comes from.

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$$4x + 2y = 10$$

$$4x + 2y - 4x = 10 - 4x$$

$$2y = -4x + 10$$

$$\frac{2y}{2} = \frac{-4x + 10}{2}$$

$$y = -2x + 5$$

Notice that we got the slope from dividing the coefficients of the variables. Specifically, if we started with the standard form equation $Ax + By = C$, we took $-A$ divided by B. In other words, we can simply look at the ratio of the coefficients in each equation. If they are the same, then the lines will have the same slope meaning it will definitely either have no solutions or infinite solutions. Look at the original system again:

$$2x + y = 5$$

$$4x + 2y = 10$$

Notice that the ratio of the coefficients, $\frac{-A}{B}$, for both equations is equal: $\frac{-2}{1} = \frac{-4}{2} = -2$. That means there are either no solutions or infinite solutions. The y-intercept will tell us which one, but remember that if they two equations are the exact same, there will be infinite solutions. Otherwise it will be no solutions.

If we divided the second equation by 2 on both sides we would get the first equation. Since the two equations would be the same, any point on the line represented by the first equation would be on the line of the second equation. That means we know there are infinite solutions and didn't have to do any work at all.

Now consider the following system. How many solutions are there?

$$3x - 2y = 5$$

$$2x - 3y = 5$$

Check the ratios of the coefficients. Notice that $\frac{-3}{-2} \neq \frac{-2}{-3}$ which means that the lines are not parallel. That tells us there is one solution, and we should use graphing, substitution, or elimination to find the solution.

Solution Steps

In essence, we follow these steps if the equations are not in slope-intercept form:

1) Make sure both equations are in standard form and check if the ratio of the coefficients are equal
    a. If the ratios are not equal, there is a single solution, and you need to solve.
    b. If the ratios are equal, then check if you can make the equations exactly the same.
        i. If the equations can be made the same, there are infinite solutions.
        ii. If the equations cannot be made the same, there are no solutions.

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Lesson 3.4

Decide if the following systems of equations have a single solution, no solutions, or infinite solutions. If it has a solution, solve the system.

1. $x + y = 1$
   $x + y = 5$

2. $2x + 3y = 7$
   $4x + 5y = 13$

3. $\frac{1}{2}x + 3y = 1$
   $x + 6y = 2$

4. $x + \frac{1}{3}y = -10$
   $3x + y = 30$

5. $2y = 6$
   $3(x + y) = 12$

6. $x + y = 2$
   $3x + 3y = 6$

7. $x + 5y = 9$
   $x + 5y = 6$

8. $2y = 5$
   $4y = 15$

9. $x + \frac{3}{5}y = 2$
   $y = -2x + 3$

10. $3x + y = 10$
    $y - 10 = -3x$

11. $3x + y = 5$
    $y = -3x + 5$

12. $6x + 4y = 10$
    $3y - 10 = -7x$

13. $2x + y = 4$
    $y - 5 = -2x$

14. $5x - 4y = 3$
    $5x = 4y - 3$

15. $7x + 5y = 3$
    $5y - 3 = -7x$

16. $\frac{2}{3}x - y = 0$
    $2x = 3y$

17. $4x = 4$
    $2x + 2y = 4$

18. $x = 2$
    $2(x + y) = 4$

19. $x + 4y = 2$
    $2(x + 4y) = 10$

20. $10x = 10 - 2y$
    $5x + y = 5$

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Write a system of equations for each situation and solve using inspection.

21. The sum of two numbers is 100. Twice the first number plus twice the second number is 200. What are the numbers?

22. The perimeter of a rectangle is 40 in. Twice the length of the rectangle is 20 minus twice the width. What are the length and width?

23. Coffee worth $2.95 a pound was mixed with coffee worth $3.50 a pound to produce a blend worth $3.30 a pound. How much of each kind of coffee was used to produce 44 pounds of blended coffee?

24. Jeri has a total of 40 pets with a total of 160 legs. If she owns only cats and dogs, how many of each does she have?

25. Pam's age plus Tom's age is 65. Twice Pam's age is equal to 130 minus twice Tom's age. How old are they?

26. The sum of two numbers is 50. Three times the first number minus three times the second number is 30. What are the numbers?

27. The perimeter of a rectangle is 30 cm. Four times the length of the rectangle is equal to 120 minus four times the width. What are the length and width?

28. A customer bought six cups of coffee and four bagels and paid $10. Another customer bought three cups of coffee and two bagels and paid $15. How much are each cup of coffee and each bagel?

29. A family went to Six Flags and bought two adult tickets and five child tickets and paid $160. A second family bought two adult tickets and eight child tickets and paid $220. How much is each adult ticket and each child ticket?

30. Jorge bought two T-shirts and four hoodies for the CMS Student Council for $80. Xavier bought one T-shirt and two hoodies for $40. How much is each T-shirt and each hoodie?

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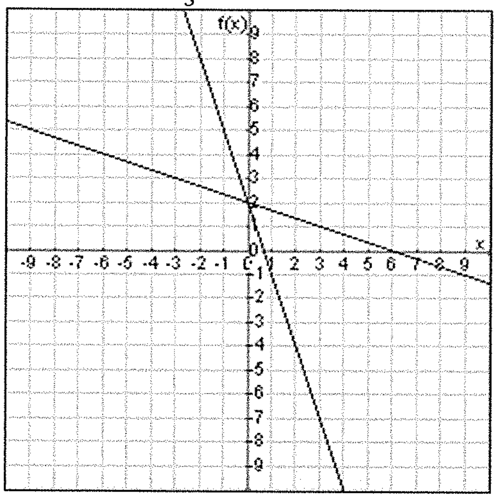
Review Unit 3: Systems

No calculator necessary. Please do not use a calculator.

Estimate the solution to the system of equations using the graph provided. Give your answer in the form of a point.

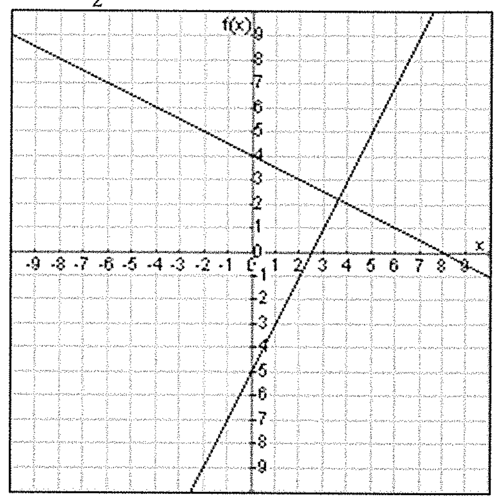
1. $y = -3x + 2$

   $y = -\frac{1}{3}x + 2$



2. $y = 2x - 5$

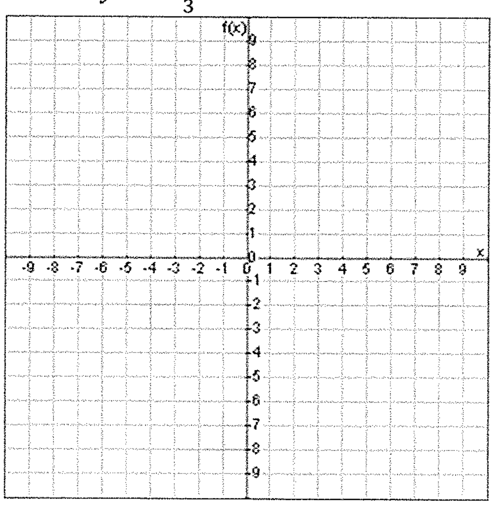
   $y = -\frac{1}{2}x + 4$



Estimate the solution to the system of equations by graphing each equation on the graph provided. Give your answer in the form of a point.

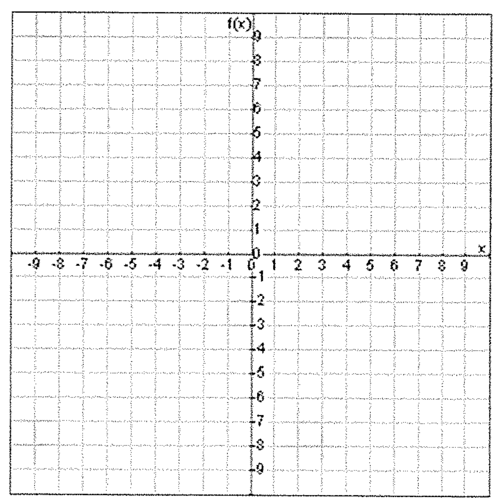
3. $y = 3x - 7$

   $y = -\frac{1}{3}x + 3$



4. $y = 2x + 7$

   $y = 3$



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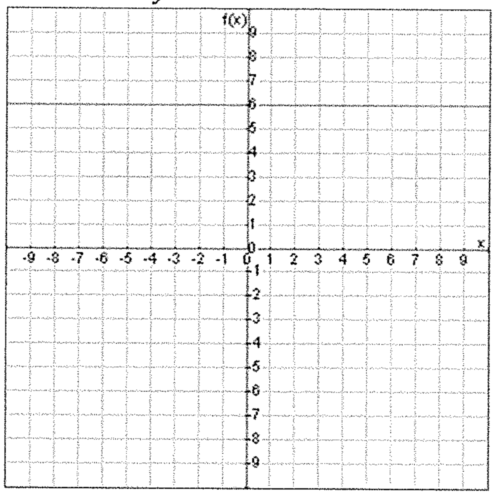
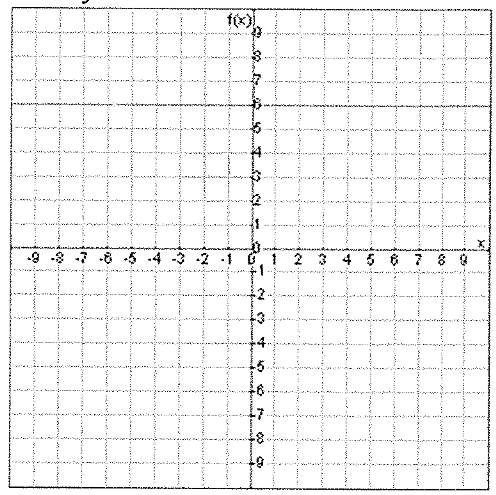
Estimate the solution to the system of equations by graphing each equation on the graph provided. Give your answer in the form of a point.

5. $-2x + 2y = 4$
   $x + y = -8$



6. $x + \frac{1}{3}y = 2$
   $-x + y = -10$



Solve the following systems of equations using any method. There could be one solution, infinite solutions, or no solution.

7. $3x + 3y = 9$
   $x + y = 3$

8. $x - 2y = 8$
   $-\frac{1}{4}x + 2y = -11$

9. $y = 5$
   $x + y = 4$

10. $6x + 2y = 8$
    $3x + y = 4$

11. $x + y = 1$
    $2x + 2y = 4$

12. $y = 3x - 5$
    $6x - 2y = 10$

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Write and solve equations for the following situations. **YOU MAY USE A CALCULATOR ON THESE!**

13. Kera sells glasses of Koolaid for $1 each and lemon shakeups for $3 each. One day she sold 10 more lemon shakeups than glasses of Koolaid, and she made a total of $190 selling. How many glasses of Koolaid and lemon shakeups did she sell?

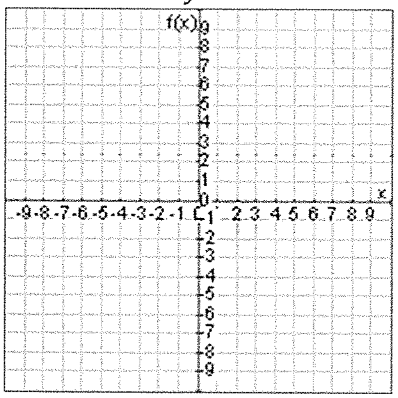
14. The perimeter of a rectangle is 14 cm. The length of the rectangle is 4 cm more than twice the width. Find the dimensions of the rectangle.

15. A 12% brine solution was mixed with a 16% brine solution to produce a 15% brine solution. How much of the 12% brine solution and how much of the 16% brine solution were used to produce 40 L of the 15% solution?

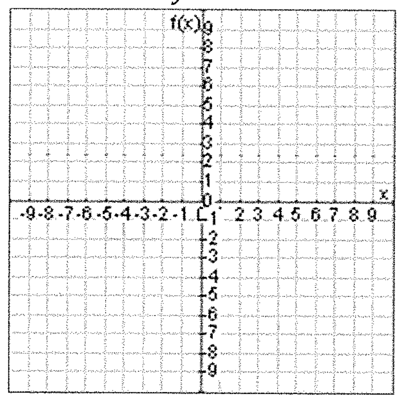
16. One customer purchased 2 lattes and 1 hot chocolate for $9. The next customer purchased 2 lattes and 3 hot chocolates for $13. How much did each latte and each hot chocolate cost?

Solve the systems of inequalities by graphing on the coordinate plane.

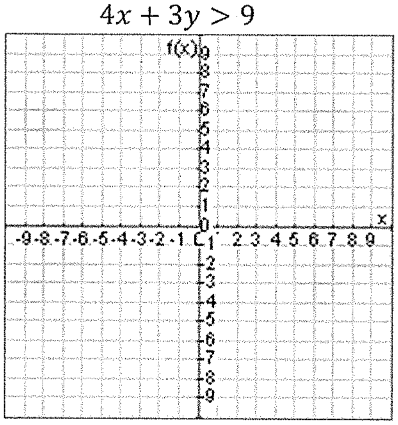
17. $3x + y > -2$
$6x + 2y < 8$



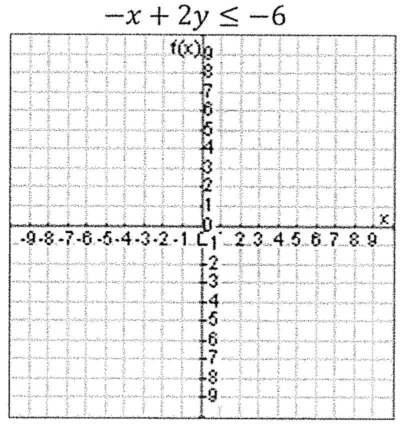
18. $x + 3y > 0$
$x - 2y > 4$



19. $-2x - 3y \leq -6$
$4x + 3y > 9$



20. $x - 3y < 0$
$-x + 2y \leq -6$



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