8th Grade Mathematics

Unit 2

Geometric Applications of Exponents

Worth County Middle School

2015-2016

STANDARDS FOR MATHEMATICAL CONTENT

Understand and apply the Pythagorean Theorem. MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| < 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| < 10$) has one solution. Evaluate square roots of perfect squares < 625 and cube roots of perfect cubes > -1000 and < 1000.

ESSENTIAL QUESTIONS

- What is the length of the side of a square of a certain area?
- What is the relationship among the lengths of the sides of a right triangle?
- How can the Pythagorean Theorem be used to solve problems?
- What is the relationship between the Pythagorean Theorem and the distance formula?
- How can I use the Pythagorean Theorem to find the length of the hypotenuse or leg of a right triangle?
- How do I know that I have a convincing argument to informally prove Pythagorean Theorem?
- What is Pythagorean Theorem and when does it apply?
- Where can I find examples of two and three-dimensional objects in the real-world?
- How does a change in any one of the dimensions of cylinder, cone, or sphere affect the volume of that cylinder, cone, or sphere?
- How does the volume of a cylinder, cone, and sphere with the same radius change if it is doubled?

CONCEPTS & SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

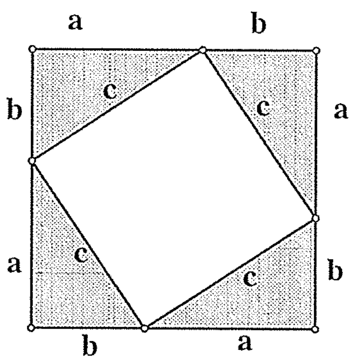
- properties of similarity, congruence, and right triangles
- understand the meaning of congruence: that all corresponding angles and sides are congruent
- two figures are congruent if they have the same shape and size
- represent radical expressions in radical form (irrational) or approximate these numbers as rational
- find square roots of perfect squares
- write a decimal approximation for an irrational number to a given decimal place
- measuring length and finding perimeter and area of quadrilaterals
- characteristics of 2-D and 3-D solids
- evaluating linear and literal equations in one variable with one solution
- properties of exponents and real numbers (commutative, associative, distributive, inverse and identity) and order of operations
- express solutions using the real number system

8.1 Pythagorean Theorem and Converse

The Pythagorean Theorem states that if a triangle is a right triangle, then the sum of the squares of the lengths of the legs equals the square of the hypotenuse lengths. That's a complicated way to say that if the legs of the triangle measure a and b and the hypotenuse measures c, then $a^2 + b^2 = c^2$. While you may have heard this in the past, we will now prove it.

Proof of the Pythagorean Theorem

There are many ways to prove the Pythagorean Theorem, but take a look at the following picture. We will refer to this for our proof.



In this picture we have a large square whose side lengths are equal to $a + b$ and an inner square whose side lengths are c. Notice that if we find the area of the large square and subtract the area of the triangles we get the area of the inner square. So let's do that algebraically.

The area of the larger square is:

$$(a+b)^2 = (a+b)(a+b) = a(a+b) + b(a+b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

The area of each triangle is $\frac{1}{2}ab$ and since there are four of them, the total area of the triangles is $2ab$.

The area of the inner square is c^2.

This means the large square minus the triangles would look like this:

$$a^2 + 2ab + b^2 - 2ab = c^2$$

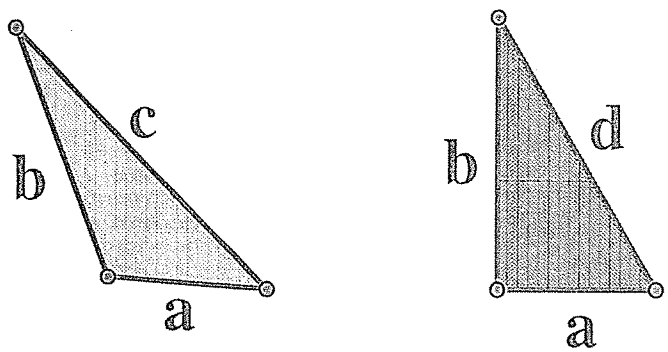
Notice that the $+2ab$ and the $-2ab$ cancel each other out (become zero), so we do get the result we expect which is that $a^2 + b^2 = c^2$.

Do a search online to see if you can find another proof for this vital theorem.

Proof of the Pythagorean Theorem Converse

The converse of the Pythagorean Theorem states that if a triangle with side lengths a, b, and c has the property that $a^2 + b^2 = c^2$, then it is a right triangle. We will now prove this.

Assume you have a triangle with side lengths a, b, and c has the property that $a^2 + b^2 = c^2$. Now construct another triangle with side lengths a and b, but make it a right triangle this time with a hypotenuse of length d. The picture would look like this with the original triangle on the left (the one that we don't know whether it is a right triangle or not) and the new triangle on the right (the one we make specifically to be a right triangle).



Since we know the Pythagorean Theorem is true, we know that $a^2 + b^2 = d^2$ which means that $d = \sqrt{a^2 + b^2}$ by taking the square root of both sides.

This means that $d = c$ since $c = \sqrt{a^2 + b^2}$ as well by the original statement that for this triangle $a^2 + b^2 = c^2$. Since all three side lengths are the same, the two triangles are congruent which means that the first triangle must be a right triangle just like the second one we made.

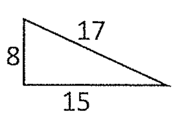
We can't use what has been called the LLP, or the Looks Like it Postulate. Just because the triangle on the left doesn't look like a right triangle, doesn't mean it actually isn't based on the facts we are given about it. The picture is inaccurate in this case.

Implications for the Pythagorean Theorem and its Converse

Now that we know both if a triangle is right then $a^2 + b^2 = c^2$ and if $a^2 + b^2 = c^2$ then the triangle is right, we can solve multiple types of problems. Given any two side lengths of a right triangle we can solve for the third side length using the Pythagorean Theorem. Given three side lengths of a triangle we can test if it's a right triangle using the Pythagorean Theorem converse.

Is it Right?

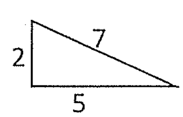
Because of the Pythagorean Converse, we can check whether a triangle is a right triangle or not. Consider the following two triangles. If their side lengths make the Pythagorean Theorem true, they are right.



$8^2 + 15^2 = 17^2$

$64 + 225 = 289$

True, so this is a right triangle.



$2^2 + 5^2 = 7^2$

$4 + 25 \neq 49$

False, $4 + 25$ is not 49, so it is not a right triangle.

Lesson 8.1

Answer the following questions.

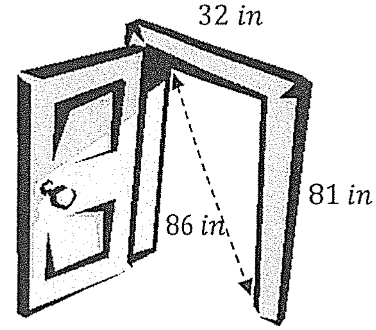
1. What is the Pythagorean Theorem in your own words?

2. What does the Pythagorean Theorem allow us to do?

3. What is the Pythagorean Theorem Converse in your own words?

4. What does the Pythagorean Theorem Converse allow us to do?

5. The door to your bathroom has never closed well. In fact, every time you try to use the bathroom, the cats bust open the door because it simply won't latch. You look at the door and it appears that the door frame is slightly tilted. The person who built your house claims that can't be true because he measured your door frame and found it be an exact right angle. He claims what you're seeing is an optical illusion.



a. Without having a protractor, what could you do to see if he is correct without having a protractor?

b. If you knew the door frame measurements were as pictured to the right, did the builder install your door frame correctly at a right angle?

6. Bob is building a triangular garden and needs fencing around it to keep the rabbits out. He has one section of fence measuring 40 ft, another measuring 42 ft, and a third measuring 58 ft. Bob says that after the fence is complete it will make a right triangle using the following argument: "First, I'll set-up the longest section of fence. Next, I'll attach the other two sections to either end of the long one. Finally, I'll swing the two shorter sections together. Since they must meet together, that makes it a right triangle."

a. Is Bob correct that the garden fence will make a right triangle?

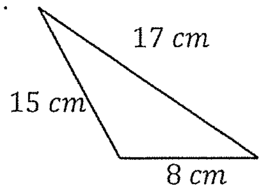
b. If so, is Bob's argument correct for why it will make a right triangle?

c. What would be a better argument?

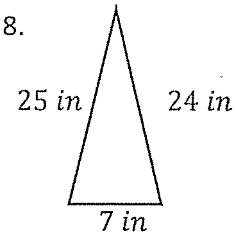
4

Determine if the following triangles are right triangles or not using the Pythagorean Theorem Converse.

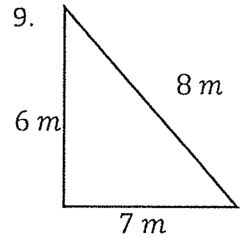
7.



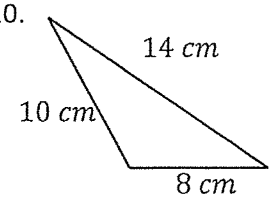
8.



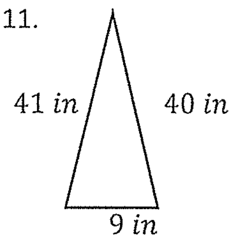
9.



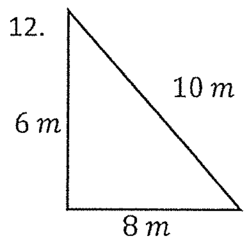
10.



11.



12.



13. $a = 12\ ft$
$b = 16\ ft$
$c = 25\ ft$

14. $a = 12\ km$
$b = 35\ km$
$c = 37\ km$

15. $a = 10\ mm$
$b = 24\ mm$
$c = 27\ mm$

16. $a = 20\ ft$
$b = 21\ ft$
$c = 29\ ft$

17. $a = 5\ km$
$b = 12\ km$
$c = 17\ km$

18. $a = 5\ mm$
$b = 12\ mm$
$c = 13\ mm$

5

NAME ______________________ DATE ______________ PERIOD ________

Reteach

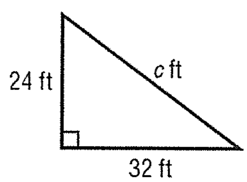
The Pythagorean Theorem

The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse for any right triangle. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. You can use the Pythagorean Theorem to find the length of a side of a right triangle if the lengths of the other two sides are known.

Examples

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

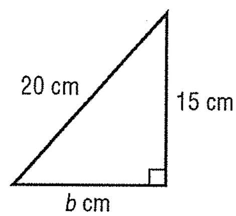
1.



$$a^2 + b^2 = c^2$$
$$24^2 + 32^2 = c^2$$
$$576 + 1{,}024 = c^2$$
$$1{,}600 = c^2$$
$$\pm\sqrt{1{,}600} = c$$
$$c = 40 \text{ or } -40$$

Length must be positive, so the length of the hypotenuse is 40 feet.

2.



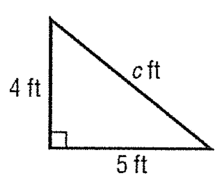
$$a^2 + b^2 = c^2$$
$$15^2 + b^2 = 20^2$$
$$225 + b^2 = 400$$
$$225 + b^2 - 225 = 400 - 225$$
$$b^2 = 175$$
$$\sqrt{b^2} = \pm\sqrt{175}$$
$$b \approx \pm 13.2$$

The length of the other leg is about 13.2 centimeters.

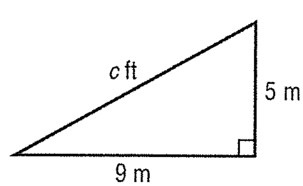
Exercises

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

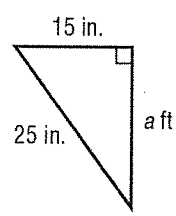
1.



2.



3.



4. $a = 7$ km, $b = 12$ km

5. $a = 10$ yd, $c = 25$ yd

6. $b = 14$ ft, $c = 20$ ft

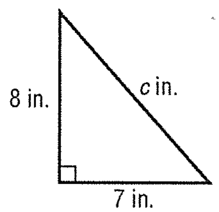
NAME ______________________ DATE ______________ PERIOD ______

Skills Practice

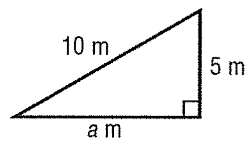
The Pythagorean Theorem

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

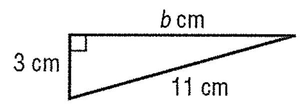
1.



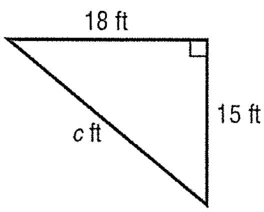
2.



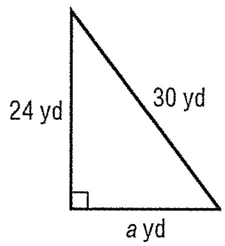
3.



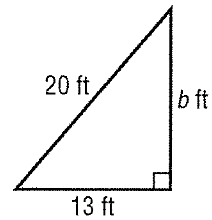
4.



5.



6.



7. $a = 1$ m, $b = 3$ m

8. $a = 2$ in., $c = 5$ in.

9. $b = 4$ ft, $c = 7$ ft

10. $a = 4$ km, $b = 9$ km

11. $a = 10$ yd, $c = 18$ yd

12. $b = 18$ ft, $c = 20$ ft

13. $a = 5$ yd, $b = 11$ yd

14. $a = 12$ cm, $c = 16$ cm

15. $b = 22$ m, $c = 25$ m

16. $a = 21$ ft, $b = 72$ ft

17. $a = 36$ yd, $c = 60$ yd

18. $b = 25$ mm, $c = 65$ mm

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

19. 10 yd, 15 yd, 20 yd

20. 21 ft, 28 ft, 35 ft

21. 7 cm, 14 cm, 16 cm

22. 40 m, 42 m, 58 m

23. 24 in., 32 in., 38 in.

24. 15 mm, 18 mm, 24 mm

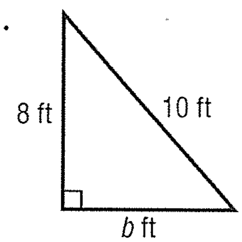
NAME ______________________ DATE ______________ PERIOD ________

Homework Practice

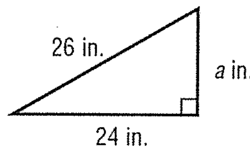
The Pythagorean Theorem

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

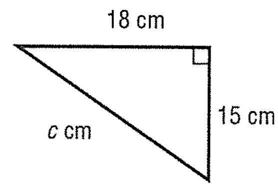
1.



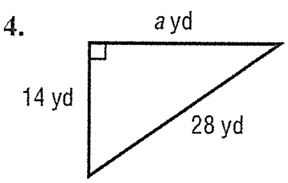
2.



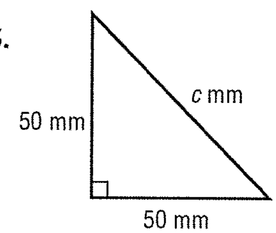
3.



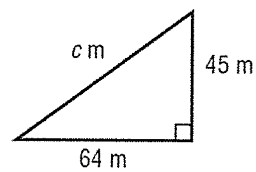
4.



5.



6.



7. *a*, 65 cm; *c*, 95 cm

8. *a*, 16 yd; *b*, 22 yd

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

9. 18 ft, 23 ft, 29 ft

10. 7 yd, 24 yd, 25 yd

11. The hypotenuse of a right triangle is 15 inches, and one of its legs is 11 inches. Find the length of the other leg.

12. A leg of a right triangle is 30 meters long, and the hypotenuse is 35 meters long. What is the length of the other leg?

13. **TELEVISIONS** The diagonal of a television measures 27 inches. If the width of a 27-inch is 22 inches, calculate its height to the nearest inch.

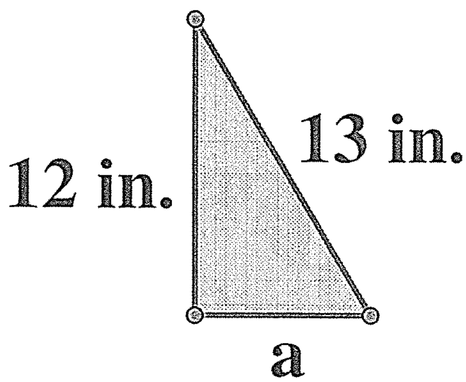
8

8.2 2D Applications of the Pythag. Theorem

Since we know that in a right triangle the statement $a^2 + b^2 = c^2$ must be true, we can now solve for any missing side length given the other two side lengths. The process of solving for a missing leg (a or b) is only slightly different from solving for a missing hypotenuse (c).

Solving for a Missing Leg

Let's first solve for a missing leg. First note that it makes no difference which leg we label as a and which leg we label as b. This is because the commutative property says that we can add in any order. In other words, whether we have $a^2 + b^2$ or $b^2 + a^2$ doesn't matter, it will always equal c^2. So if we are missing the length of a leg, it might be easiest to always assume it is a that is missing.



Given the fact that this is a right triangle, we can solve for the missing leg length, a. Just substitute everything we know into the Pythagorean Formula. We know that the hypotenuse length, c, is 13 inches and that the other leg length, b, is 12 inches.

$$a^2 + b^2 = c^2$$

$$a^2 + (12)^2 = (13)^2$$

Now go ahead and multiple out those exponents to get the following statement:

$$a^2 + 144 = 169$$

Notice this is a two-step equation where a is being squared and then increased by 144. Applying inverse operations, we know we should subtract 144 from both sides and then take the square root. That looks like this:

$$a^2 + 144 = 169$$

$$-144 \quad -144$$

$$a^2 = 25$$

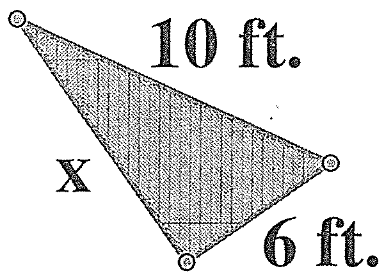
$$\sqrt{a^2} = \sqrt{25}$$

$$a = 5$$

We have just proved that the missing side length must be 5 inches.

9

Sometimes the missing side length will be labeled with a different variable just to throw us off. Just remember that the legs are always a and b in the Pythagorean Formula and that c, or the hypotenuse, is always the longest side length. For example, in the following picture which are the legs and which is the hypotenuse?



The hypotenuse is always opposite (or across from) the right angle and is the longest side. So the hypotenuse in this picture is 10 ft. That means that the 6 ft and the x must be the two sides. Notice that the legs can also be identified by the fact that they are the sides that make up the right angle. Now substitute into the Pythagorean Formula to solve for x.

$$x^2 + (6)^2 = (10)^2$$

$$x^2 + 36 = 100$$

$$-36 \quad -36$$

$$x^2 = 64$$

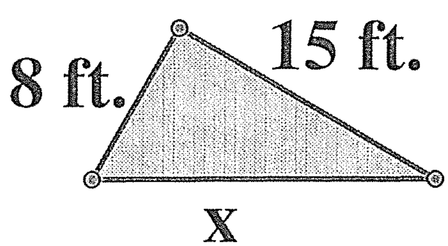
$$\sqrt{x^2} = \sqrt{64}$$

$$x = 8$$

So we know that the missing side length is 8 ft. in this particular triangle.

Solving for a Missing Hypotenuse

Let's now solve for a missing hypotenuse. Remember that the hypotenuse is always the longest side and the side opposite the right angle. Take a look at this example.



Note that 8 ft. and 15 ft. must the lengths of the legs since they make up the right angle. That means that x in this case is the missing hypotenuse. Plugging those values into the Pythagorean Formula yields the following:

$$(8)^2 + (15)^2 = x^2$$

$$64 + 225 = x^2$$

Be careful at this point. Many students mistakenly try to subtract either 64 or 225 from both sides, but that is not accurate. We always combine like terms before using inverse operations, and in this case we still need to combine the 64 + 225 to get 289. So our next steps should look like this:

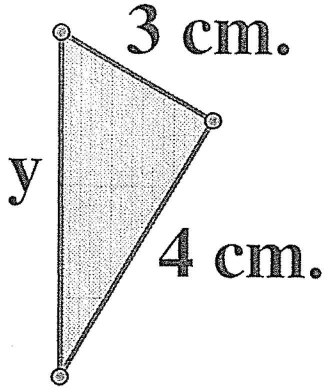
$$289 = x^2$$

$$\sqrt{289} = \sqrt{x^2}$$

$$17 = x$$

This means that the missing hypotenuse length is 17 feet. Note that the only inverse operation we needed to apply in this case was the square root.

Let's look at one more example of solving for a missing hypotenuse. Consider the following picture.



Note that y is the hypotenuse in this case because the sides with lengths 3 and 4 make up the right angle. Plug these values into the Pythagorean Formula.

$$(3)^2 + (4)^2 = y^2$$

$$9 + 16 = y^2$$

$$25 = y^2$$

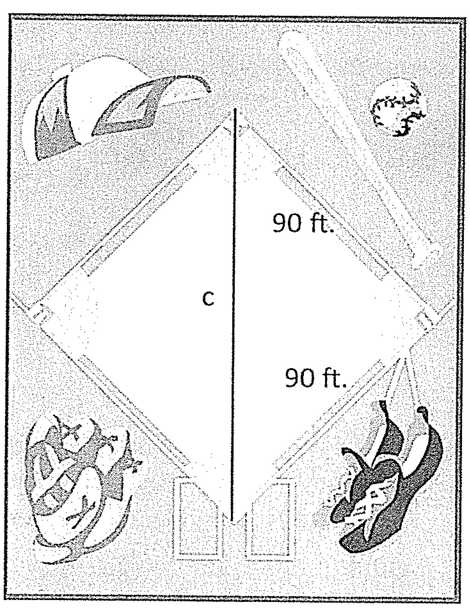
$$\sqrt{25} = \sqrt{y^2}$$

$$5 = y$$

So the hypotenuse has a length of 5 centimeters in this case.

Pythagorean Theorem Word Problems

The use of the Pythagorean Theorem can applied to word problems just as easily. For example, if we know that it is 90 feet from home plate to first base and 90 feet from first base to second base, how far would the catcher have to throw the baseball to get a runner out who is stealing second base? The best tip to give for solving word problems like this is to draw a picture.



In this case, note that the distance from second base to home plate is the hypotenuse of the triangle. That means that the 90 feet distances are the legs. We can now solve as follows.

$$(90)^2 + (90)^2 = c^2$$

$$8100 + 8100 = c^2$$

$$16200 = c^2$$

$$\sqrt{16200} = \sqrt{c^2}$$

$$127.3 \approx c$$

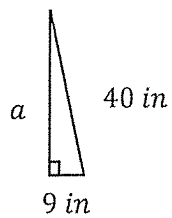
For this problem, there was no exact square root. That means that $\sqrt{16200}$ is irrational and it's probably best to estimate this number. Our answer is approximated to the nearest one decimal place giving us about 127.3 feet. So the catcher would have to throw just over 127 feet to get out the runner trying to steal second base.

11

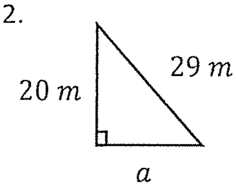
Lesson 8.2

Find the length of the missing side of each right triangle. Round your answers to three decimal places if necessary.

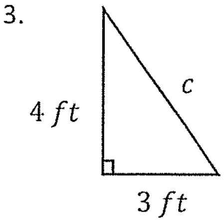
1.



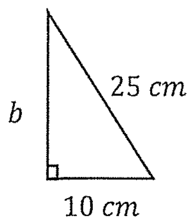
2.



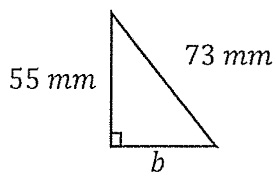
3.



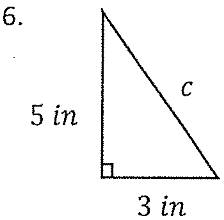
4.



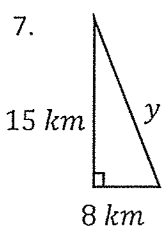
5.



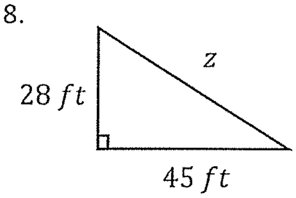
6.



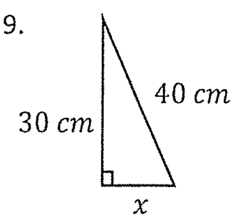
7.



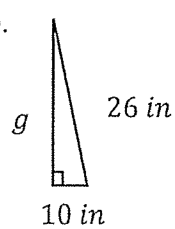
8.



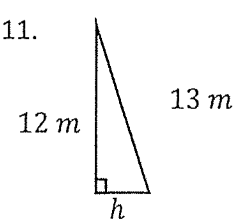
9.



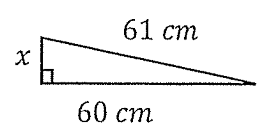
10.



11.

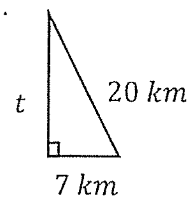


12.

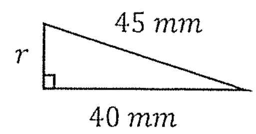


12

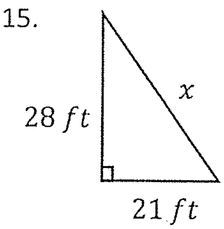
13.



14.



15.



Solve the following problems. Round your answers to the nearest whole number when necessary.

16. You're locked out of your house, and the only open window is on the second floor 25 feet above the ground. There are bushes along the side of the house that force you to put the base of the ladder 7 feet away from the base of the house. How long of a ladder will you need to reach the window?

17. Shae takes off from her house and runs 3 miles north and 4 miles west. Tired, she wants to take the shortest route back. How much farther will she have to run if she heads straight back to her house?

18. Televisions are advertised by the length of their diagonals. If a 42 inch television measures 18 inches high, how wide is the television?

19. A soccer field is 100 yards by 60 yards. How long is the diagonal of the field?

20. Leonard walks 14 meters south and 48 meters east to get to school. If he takes the straight path home after school, how far will he have to walk?

21. You place a 24 foot ladder 10 feet away from the house. The top of the ladder just reaches a window on the second floor. How high off the ground is the window?

22. The dimensions of a basketball court are 74 feet and 42 feet. What is the length of the diagonal of the court?

13

23. Televisions are advertised by the length of their diagonals. If a TV measures 22 inches high and 45 inches wide, by what size will the TV be advertised.

24. A rectangular garden measures 5 feet wide by 12 feet long. If a hose costs $5 per foot, how much would it cost to place a hose through the diagonal of the garden?

25. A football field is 160 feet wide and 360 feet long. The coach wants to put spray paint along the diagonal of the field. If the spray paint costs approximately $1 per foot of coverage, how much should the coach budget for spray paint?

26. A rectangular park measures 8 miles long by 6 miles wide. The park director wants to put a fence along both sides of the trail that runs diagonally through the park. If the fence costs $150 per mile, how much will it cost to buy the fence?

27. A rectangular pool has a diagonal of 17 yards and a length of 15 yards. If the paint costs $2 per yard of coverage, how much will it cost the owner to paint the width of both ends of the pool?

28. A rectangular dog pen is 3 meters by 4 meters. If a chain costs $1.75 per meter, how much would it cost to put a chain along the diagonal of the pen?

29. Architects built a doorway that was 4 feet wide by 7 feet tall. The diagonal measured 7.3 feet. Are the angles in the doorway right angles?

30. A rectangular garden measures 3 meters wide by 4 meters long. The diagonal of the garden measures 5 meters. Are the angles in the garden right angles?

14

NAME ______________________ DATE ______________________ PERIOD ____________

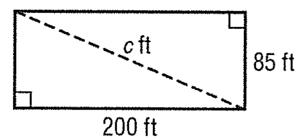
Reteach

Use the Pythagorean Theorem

The Pythagorean Theorem can be used to solve a variety of problems.

Example

A professional ice hockey rink is 200 feet long and 85 feet wide. What is the length of the diagonal of the rink?



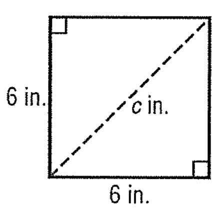
$a^2 + b^2 = c^2$	The Pythagorean Theorem
$200^2 + 85^2 = c^2$	Replace *a* with 200 and *b* with 85.
$40{,}000 + 7{,}225 = c^2$	Evaluate 200^2 and 85^2.
$47{,}225 = c^2$	Add 40,000 and 7,225.
$\sqrt{47{,}225} = c^2$	Definition of square root
$\sqrt{217.3} \approx c$	Use a calculator.

The length of the diagonal of an ice hockey rink is about 217.3 feet.

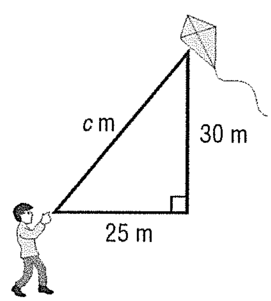
Exercises

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

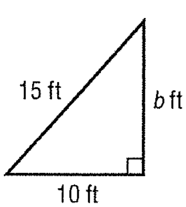
1. What is the length of the diagonal?



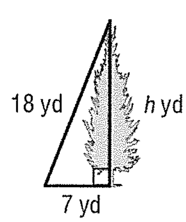
2. How long is the kite string?



3. What is the height of the ramp?



4. How tall is the tree?



15

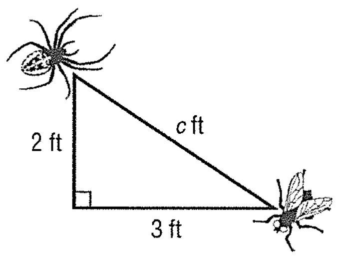
NAME ______________________ DATE ______________ PERIOD ________

Skills Practice

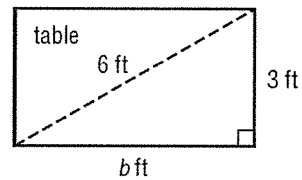
Use the Pythagorean Theorem

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

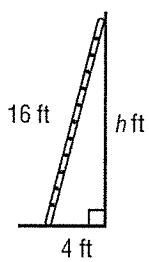
1. How far apart are the spider and the fly?



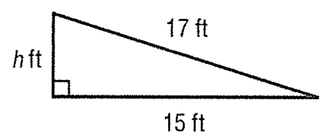
2. How long is the tabletop?



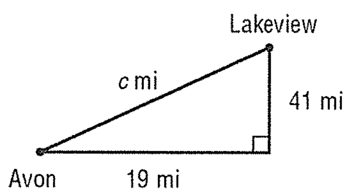
3. How high will the ladder reach?



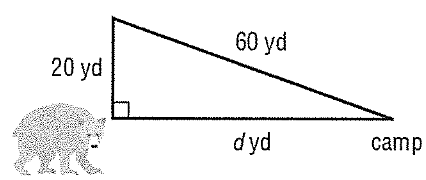
4. How high is the ramp?



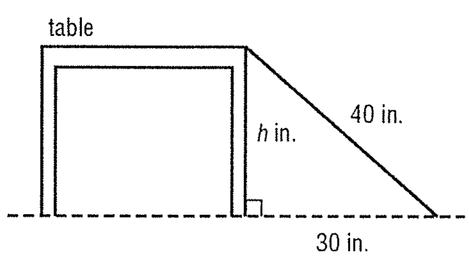
5. How far apart are the two cities?



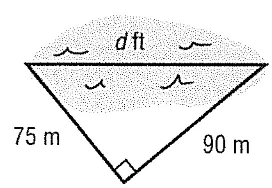
6. How far is the bear from camp?



7. How tall is the table?



8. How far is it across the pond?



16

NAME ______________________ DATE ________________ PERIOD ________

Problem-Solving Practice

Use the Pythagorean Theorem

1. **RECREATION** A pool table is 8 feet long and 4 feet wide. How far is it from one corner pocket to the diagonally opposite corner pocket? Round to the nearest tenth.

2. **TRIATHLON** The course for a local triathlon has the shape of a right triangle. The legs of the triangle consist of a 4-mile swim and a 11 mile run. The hypotenuse of the triangle is the biking portion of the event. How far is the biking part of the triathlon? Round to the nearest tenth if necessary.

3. **LADDER** A ladder 17 feet long is leaning against a wall. The bottom of the ladder is 8 feet from the base of the wall. How far up the wall is the top of the ladder? Round to the nearest tenth if necessary.

4. **TRAVEL** Tara drives due north for 22 miles then east for 11 miles. How far is Tara from her starting point? Round to the nearest tenth if necessary.

5. **FLAGPOLE** A wire 31 feet long is stretched from the top of a flagpole to the ground at a point 15 feet from the base of the pole. How high is the flagpole? Round to the nearest tenth if necessary.

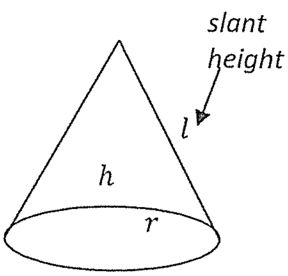
6. **ENTERTAINMENT** Isaac's television is 25 inches wide and 18 inches high. What is the diagonal size of Isaac's television? Round to the nearest tenth if necessary.

8.3 3D Applications of the Pythag. Theorem

Now that we know how to use the Pythagorean Theorem to find either a missing leg or missing hypotenuse, we can move this concept into three-dimensional concepts. Let's look at some examples.

Regular Cones and Pyramids

In both cones and pyramids we can use the Pythagorean Theorem to find the height or slant height. We can also find the radius in a cone or the side length of the base of a pyramid. First look at this cone.



The height and radius are fairly obvious, but the slant height might be new vocabulary. The slant height, usually referred to as l in problems, is the height from the outer bottom edge of the cone up to the tip. It is not the actual height because it is not perpendicular to the base. Notice that each of these three variables form a right triangle. Therefore if we know two of them we can find the other one.

For example, assume that $r = 5\ in.$ and $h = 12\ in.$ What is l, or the slant height, in inches?

$$a^2 + b^2 = c^2$$

$$r^2 + h^2 = l^2$$

$$5^2 + 12^2 = l^2$$

$$25 + 144 = l^2$$

$$169 = l^2$$

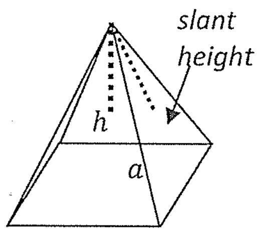
$$\sqrt{169} = \sqrt{l^2}$$

$$13\ in. = l$$

Why is this useful? Knowing each dimension allows us to find the volume or surface area of the shape. For example, the formula for the surface of a cone is $SA = \pi r^2 + \pi r l$. Now that we know $l = 13\ in.$ we can find the surface area is $SA = 25\pi + 65\pi = 90\pi\ in^2$.

18

Let's look at a pyramid missing its height.



Assume that the base of the pyramid (bottom) is a square with side length of 12 $cm.$ and the slant height is 10 $cm.$ What is the height of the pyramid? Since we have the side length of the square, we only need half of that to form the short leg of our right triangle, or 6 $cm.$ This gives us the following:

$$a^2 + b^2 = c^2$$

$$a^2 + h^2 = l^2$$

$$6^2 + h^2 = 10^2$$

$$36 + h^2 = 100$$

$$-36 \qquad -36$$

$$h^2 = 64$$

$$\sqrt{h^2} = \sqrt{64}$$

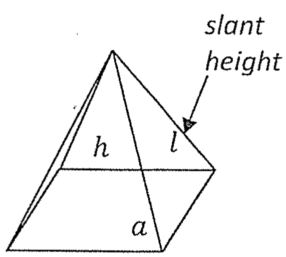
$$h = 8\ cm.$$

Again, we can now answer further questions about this shape like finding the volume. The volume of a regular pyramid uses the formula $V = \frac{1}{3}Bh$ where B is the area of the base shape (the square). This means that the volume is $\frac{1}{3}(144)(8) = 384\ cm^3$.

19

Lesson 8.3

Use the picture below to find information about the pyramid with a square base in problems 1-14. Round your answers to three decimal places if necessary.



1. The pyramid has a square base that is 70 ft on each side. The slant height is 37 ft. What is h, the height of the pyramid?

2. The pyramid has a square base that is 120 in on each side. The slant height is 61 in. What is h, the height of the pyramid?

3. The pyramid has a square base that is 50 m on each side. The slant height is 30 m. What is h, the height of the pyramid?

4. The pyramid has a square base that is 14 cm on each side. The slant height is 25 cm. What is h, the height?

5. The pyramid has a square base that is 14 cm on each side. The height is 24 cm. What is l, the slant height?

6. The pyramid has a square base that is 24 ft on each side. The height is 5 ft. What is l, the slant height?

7. The pyramid has a square base that is 70 mm on each side. The height is 10 mm. What is l, the slant height?

8. The pyramid has a square base that is 26 ft on each side. The height is 82 ft. What is l, the slant height?

9. The height of the pyramid is 15 cm, and the slant height is 39 cm. Find the value of a in the diagram.

10. The height of the pyramid is 80 in, and the slant height is 82 in. Find the value of a in the diagram.

11. The slant height is 17 ft and the height is 8 ft. What is s, the side length of the base?

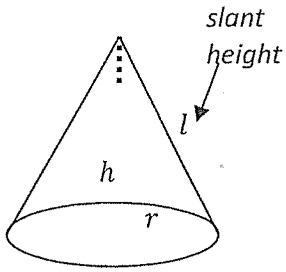
12. The slant height is 10 cm and the height is 8 cm. What is s, the side length of the base?

13. The slant height is 26 mm and the height is 10 mm. What is s, the side length of the base?

14. The slant height is 50 ft and the height is 32 ft. What is s, the side length of the base?

20

Use the picture below to find information about the pyramid in problems 15-26. Round your answers to three decimal places if necessary.



15. The cone has a radius of $12\ cm$ and a height of $5\ cm$. What is l, the slant height of the cone?

16. The cone has a radius of $15\ mm$ and a height of $8\ mm$. What is l, the slant height of the cone?

17. The cone has a radius of $24\ in$ and a height of $70\ in$. What is l, the slant height of the cone?

18. The cone has a radius of $40\ cm$ and a height of $42\ cm$. What is l, the slant height of the cone?

19. The cone has a radius of $30\ ft$ and a slant height of $34\ ft$. What is h, the height of the cone?

20. The cone has a radius of $33\ m$ and a slant height of $65\ m$. What is h, the height of the cone?

21. The cone has a radius of $16\ in$ and a slant height of $20\ in$. What is h, the height of the cone?

22. The cone has a radius of $30\ cm$ and a slant height of $50\ cm$. What is h, the height of the cone?

23. The cone has a height of $16\ cm$ and a slant height of $65\ cm$. What is r, the radius of the cone?

24. The cone has a height of $48\ ft$ and a slant height of $50\ ft$. What is r, the radius of the cone?

25. The cone has a height of $4\ in$ and a slant height of $6\ in$. What is r, the radius of the cone?

26. The cone has a height of $14\ cm$ and a slant height of $55\ cm$. What is r, the radius of the cone?

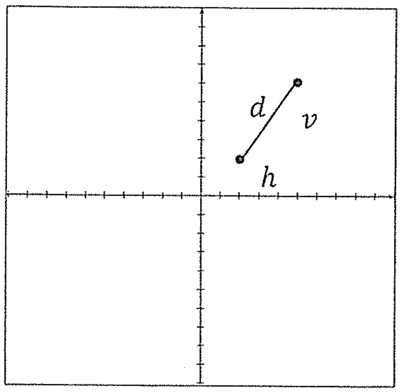
21

8.4 The Distance Between Points

A final application of the Pythagorean Theorem is on the coordinate plane. We can easily find the distance between two points vertically or horizontally on a coordinate plane just by counting, but finding the exact distance diagonally we have not been able to do until now.

The Distance between Any Two Points

On a coordinate plane, we can now find the distance between any two points by drawing in a right triangle and using the Pythagorean Theorem. Consider the following example:



Notice that if we want to find the distance between these two points, (2,2) and (5,6), we need to find the length of d. Also note that h is the horizontal distance between the points and v is the vertical distance between the points. With all those values we now have a right triangle and can use the Pythagorean Theorem as follows:

$$h^2 + v^2 = d^2$$

$$3^2 + 4^2 = d^2$$

$$9 + 16 = d^2$$

$$25 = d^2$$

$$5 = d$$

So we know that the distance between these points is five units. While this is easy to see when drawn out on the coordinate plane, there are times when we are given the two points without a picture. In that case, we have two options. We can either draw the points on the coordinate plane as above, or we can find the horizontal and vertical distance between the points in another way.

The Distance without a Coordinate Plane

To do this without graphing, we realize that the horizontal distance between two points is the difference in their x values. Why is this? Similarly, the vertical distance between two points is the difference in their y values. Again, can you explain why?

So let's look at our two points again, (2,2) and (5,6). The horizontal distance would be the difference between 2 and 5. Since difference means subtract, we can take $5 - 2 = 3$ to find the horizontal distance is 3. Similarly we can subtract the y values to get $6 - 2 = 4$ meaning a vertical distance of 4. We can then plug in 3 and 4 into the Pythagorean Theorem and solve exactly as above.

22

Enrichment: The Distance Formula

Using the information above, how would we find the distance between two generic points? We typically represent generic points with the notation of (x_1, y_1) and (x_2, y_2). So what would the horizontal and vertical distance between these two points be?

Horizontal distance: $h = x_2 - x_1$

Vertical distance: $v = y_2 - y_1$

Finally, let's substitute these into the Pythagorean Theorem of $h^2 + v^2 = d^2$ as follows and then solve for d since d is the actual distance between the points.

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{d^2}$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$

The final result is what is known as the distance formula. Let's use this formula to find the distance between the points $(-3, 4)$ and $(3, -4)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3 - (-3))^2 + ((-4) - 4)^2}$$

$$d = \sqrt{(6)^2 + (-8)^2}$$

$$d = \sqrt{36 + 64}$$

$$d = \sqrt{100}$$

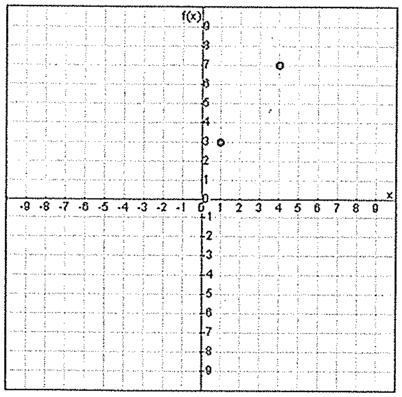
$$d = 10$$

We see that the distance between those two points is ten units. While the distance formula works, it is often easier to simply visualize the horizontal and vertical distance between two points mentally or on a coordinate plane. The distance formula is basically a fancy way to use the Pythagorean Formula and is meant for enrichment only.

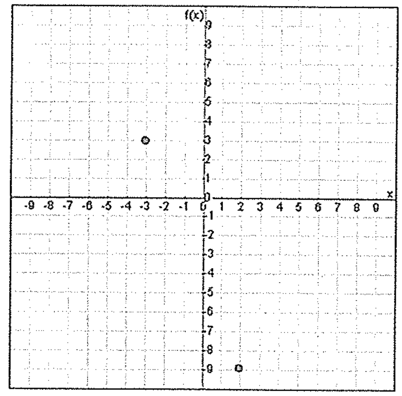
Lesson 8.4

Determine the distance between the given points. Round your answers to three decimal places if necessary.

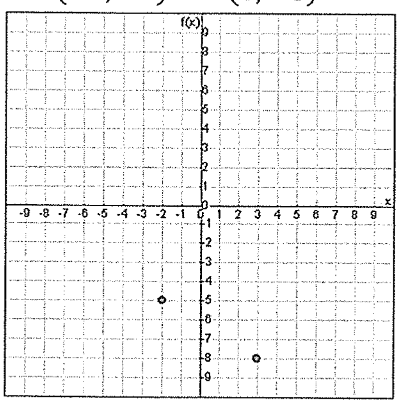
1. $(1, 3)$ and $(4, 7)$



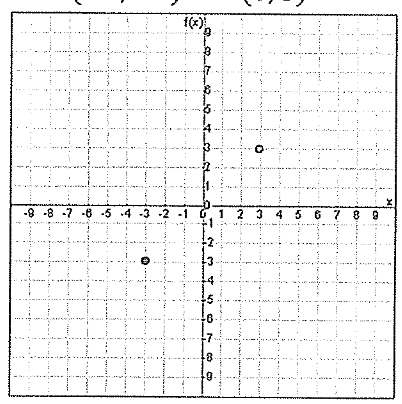
2. $(-3, 3)$ and $(2, -9)$



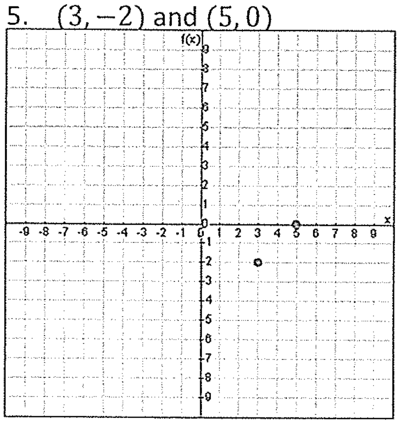
3. $(-2, -5)$ and $(3, -8)$



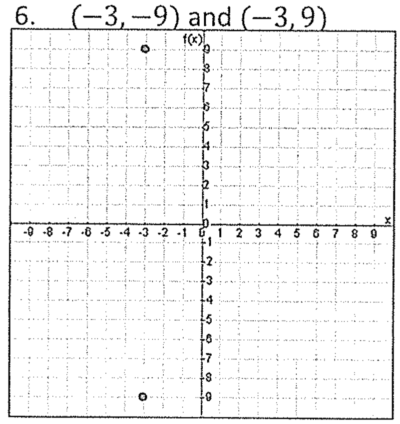
4. $(-3, -3)$ and $(3, 3)$



5. $(3, -2)$ and $(5, 0)$



6. $(-3, -9)$ and $(-3, 9)$



24

7. $(2, 1)$ and $(3, -3)$

8. $(4, -2)$ and $(7, 2)$

9. $(1, 1)$ and $(7, 9)$

10. $(-8, 2)$ and $(6, 2)$

11. $(-4, 6)$ and $(6, 2)$

12. $(2, 4)$ and $(5, -2)$

13. $(-5, -3)$ and $(6, 6)$

14. $(-5, 4)$ and $(7, 3)$

15. $(-9, -3)$ and $(-4, 4)$

16. $(2, -4)$ and $(5, 4)$

17. $(0, 7)$ and $(4, 2)$

18. $(-8, 7)$ and $(7, -5)$

NAME ______________________ DATE ______________ PERIOD ________

Reteach

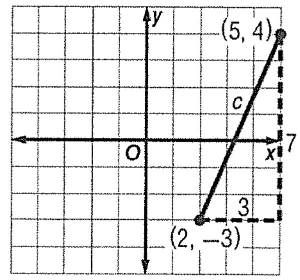
Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

Example

Graph the ordered pairs (2, –3) and (5, 4). Then find the distance *c* between the two points.

$a^2 + b^2 = c^2$	The Pythagorean Theorem
$3^2 + 7^2 = c^2$	Replace *a* with 3 and *b* with 7.
$58 = c^2$	$3^2 + 7^2 = 9 + 49$, or 58.
$\pm\sqrt{58} = \sqrt{c}$	Definition of square root
$\pm 7.6 \approx c$	Use a calculator.

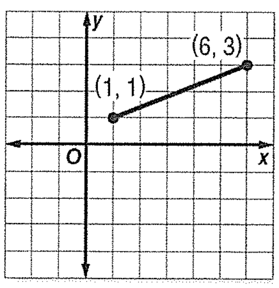


The points are about 7.6 units apart.

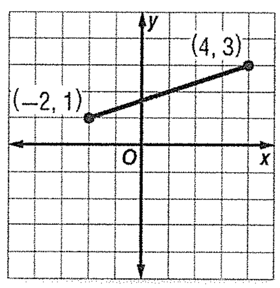
Exercises

Find the distance between each pair of points. Round to the nearest tenth if necessary.

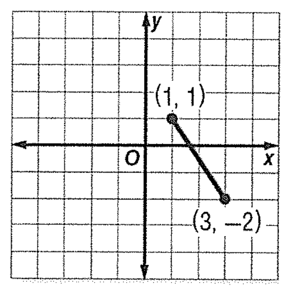
1.



2.

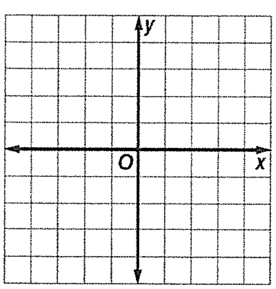


3.

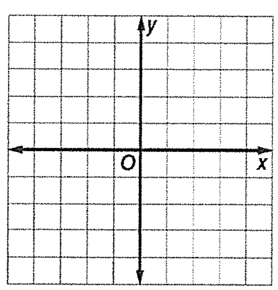


Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

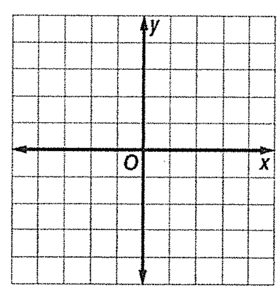
4. (4, 5), (0, 2)



5. (0, –4), (–3, 0)



6. (–1, 1), (–4, 4)



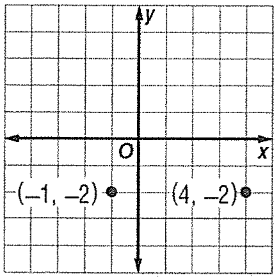
NAME ______________________ DATE ______________ PERIOD ________

Skills Practice

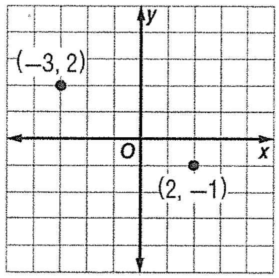
Distance on the Coordinate Plane

Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth if necessary.

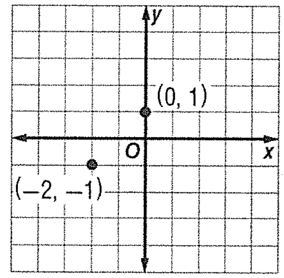
1.



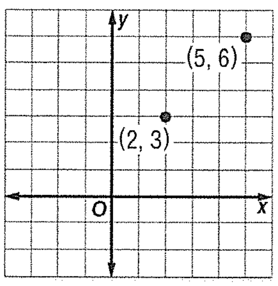
2.



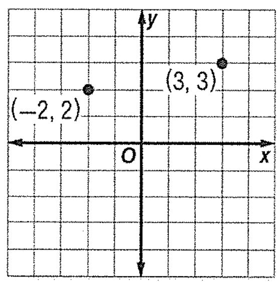
3.



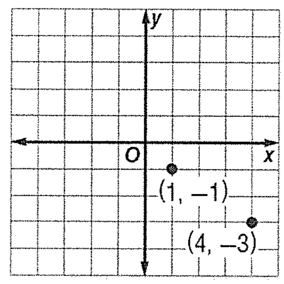
4.



5.

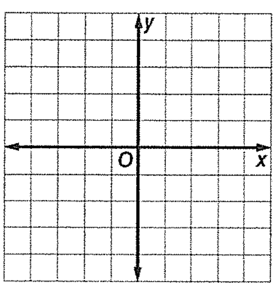


6.

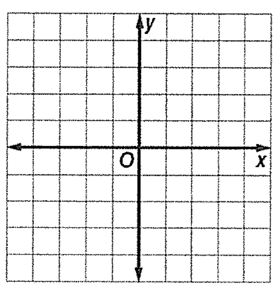


Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

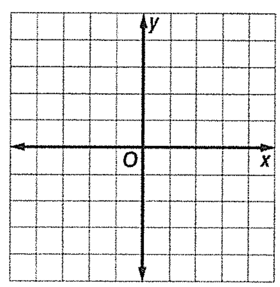
7. (–3, 0), (3, –2)



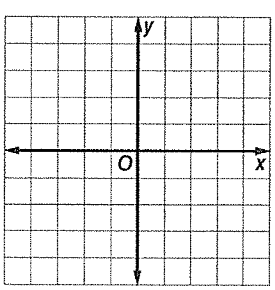
8. (–4, –3), (2, 1)



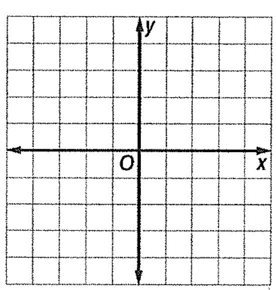
9. (0, 2), (5, –2)



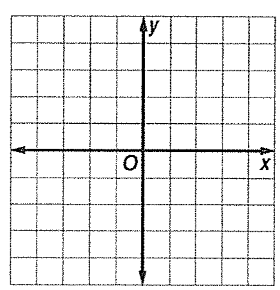
10. (–2, 1), (–1, 2)



11. (0, 0), (–4, –3)



12. (–3, 4), (2, –3)



27

NAME ______________________ DATE ______________ PERIOD ________

Problem-Solving Practice

Distance on the Coordinate Plane

1. ARCHAEOLOGY An archaeologist at a dig sets up a coordinate system using string. Two similar artifacts are found—one at position (1, 4) and the other at (5, 2). How far apart were the two artifacts? Round to the nearest tenth of a unit if necessary.	**2. GARDENING** Vega set up a coordinate system with units of feet to locate the position of the vegetables she planted in her garden. She has a tomato plant at (1, 3) and a pepper plant at (5, 6). How far apart are the two plants? Round to the nearest tenth if necessary.
3. CHESS April is an avid chess player. She sets up a coordinate system on her chess board so she can record the position of the pieces during a game. In a recent game, April noted that her king was at (4, 2) at the same time that her opponent's king was at (7, 8). How far apart were the two kings? Round to the nearest tenth of a unit if necessary.	**4. MAPPING** Cory makes a map of his favorite park, using a coordinate system with units of yards. The old oak tree is at position (4, 8) and the granite boulder is at position (–3, 7). How far apart are the old oak tree and the granite boulder? Round to the nearest tenth if necessary
5. TREASURE HUNTING Taro uses a coordinate system with units of feet to keep track of the locations of any objects he finds with his metal detector. One lucky day he found a ring at (5, 7) and an old coin at (10, 19). How far apart were the ring and coin before Taro found them? Round to the nearest tenth if necessary.	**6. GEOMETRY** The coordinates of points A and B are (–7, 5) and (4, –3), respectively. What is the distance between the points, rounded to the nearest tenth?
7. GEOMETRY The coordinates of points A, B, and C are (5, 4), (–2, 1), and (4, –4), respectively. Which point, B or C, is closer to point A?	**8. THEME PARK** Bryce is looking at a map of a theme park. The map is laid out in a coordinate system. Bryce is at (2, 3). The roller coaster is at (7, 8), and the water ride is at (9, 1). Is Bryce closer to the roller coaster or the water ride?

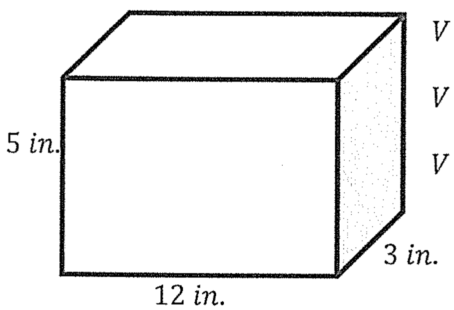
28

8.5 Volume of Rounded Objects

A basic definition of **volume** is how much space an object takes up. Since this is a three-dimensional measurement, the unit is usually cubed. For example, we might talk about how many cubic feet of water are in a pool (or ft^3) or how many cubic millimeters of ink are in an ink pen (or mm^3). Let's explore some previously learned concepts about volume before diving into cylinders.

Volume Basics

Perhaps the most recognizable formula for volume comes from a rectangular prism. This easily remembered formula is $V = lwh$, or the volume is the length times the width times the height. So in the example below, we see that the volume is $180\ in^3$.



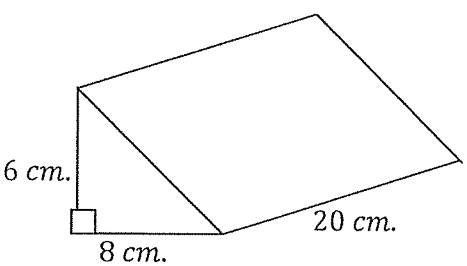
$V = lwh$

$V = 12 * 3 * 5$

$V = 180\ in^3$

This is a nice neat formula, but unfortunately it promotes the idea that all we do to find the volume is multiply all the numbers we see. This simply isn't true, so we need to know where this formula came from.

Hopefully the idea of length times width sounds familiar. We should recognize that as the area formula for a rectangle, but which rectangle specifically on our rectangular prism above? It is the bottom face of the prism, the 12 by 3 rectangle. This bottom face is known as the **base** of the prism. The base shapes of a prism are the shapes that are congruent and parallel, or the top and bottom in our prism above. This means that we could rewrite the volume formula to say $V = Bh$ or volume equals the area of the base times the height of the prism.



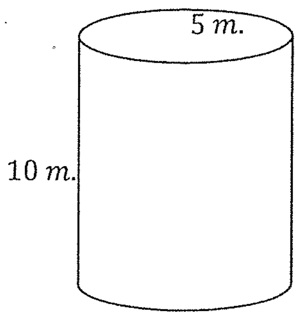
This formula is much more useful because it will work for all prisms. For example, consider the triangular prism below. In this case the base shape is a triangle. Since the **height** of a prism is the perpendicular distance between the bases, we know the height is 20 cm. So we apply our formula of $V = Bh$ by finding the area of the triangle and multiplying by 20, the height.

$$V = Bh = \left(\frac{8 * 6}{2}\right)(20) = (24)(20) = 480\ cm^3$$

In essence, what we should take from this is that the volume is like taking the base shape and stacking up more and more of those shapes until it hits the height of the prism. In other words, $V = Bh$ is a very nice and widely applicable formula.

Volume of a Cylinder

So let's take that concept of the area of the base shape multiplied by the height and transfer it to the cylinder. While the cylinder is not a prism, it is similar. The base is a circle, which we know the area to be πr^2, and the height is the distance between the circles. This means that we can still use the formula $V = Bh$ or in this case, $V = \pi r^2 h$.



$V = \pi r^2 h$

$V = \pi(5)^2(10)$

$V = \pi(25)(10)$

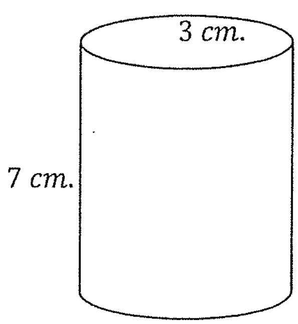
$V = \pi(250)$

$V = 250\pi \approx 250 * 3.14 \approx 785\ m^3$

This means that this particular cylinder takes up approximately 785 cubic meters of space. Another way to think about it is that about 785 little cubes that are a meter on each side would fit inside this cylinder.

In Terms of Pi

Typically it makes sense to plug in the approximation of 3.14 for pi. This gives us an idea of the actual number or size we're dealing with. However, sometimes a problem will ask for an answer in terms of pi. That means to not actually plug in 3.14 for pi. Just leave pi in the answer. So in the above example, the volume would just be $250\pi\ m^3$. Let's look at another example.



$V = \pi r^2 h$

$V = \pi(3)^2(7)$

$V = \pi(9)(7)$

$V = \pi(63)$

$V = 63\pi\ cm^3$

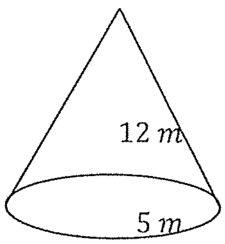
So our final answer in terms of pi is $63\pi\ cm^3$. Of course we can always plug in 3.14 for pi to get an approximate answer which in this case is $\approx 197.82\ cm^3$.

30

Volume of a Cone

Cones are similar to prisms and therefore we will use a variant of the $V = Bh$ formula. In fact, the cone is nearly the same as a cylinder except that a cone only has one base shape. At the top of the cone is a vertex instead of a second congruent circle. This means that a cone with the same exact circular base and height as a cylinder will hold less. The question is how much less?

We may recall from previous courses that the volume of a pyramid uses the formula $V = \frac{1}{3}Bh$. Since a cone is very similar to a pyramid, it would be reasonable to expect to use the same formula. It turns out this is correct. The actual proof for this takes some calculus, so we'll take it on faith for right now. Could you design an experiment to gather some evidence that this formula will work for a cone?



Let's find the volume of this cone.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(5)^2(12)$$

$$V = \frac{1}{3}\pi(25)(12)$$

$$V = \frac{1}{3}\pi(300)$$

$$V = 100\pi \approx 314\, m^3$$

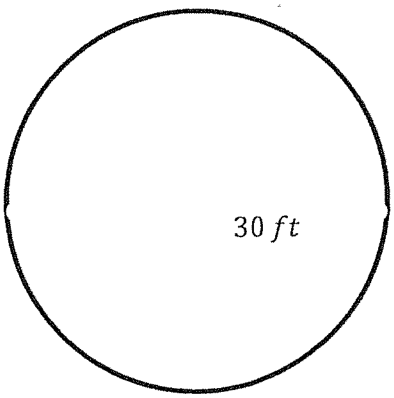
Again we can leave our answer in terms of pi or use 3.14 to approximate the answer.

31

Volume of a Sphere

While pyramids and cones share a volume formula, spheres have their own. Spheres use the formula $V = \frac{4}{3}\pi r^3$. Applying this formula is similar to applying the previous volume formulas. In the biz we call this "plug and chug" math.

Let's find the volume of this sphere.



$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi(30)^3$$

$$V = \frac{4}{3}\pi(27000)$$

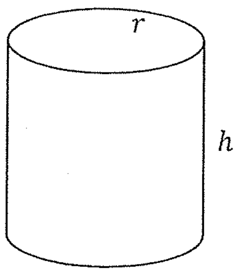
$$V = 36000\pi \approx 113040\ ft^3$$

Keep in mind that we are still following order of operations. So we take the radius cubed first. After that we apply the commutative property and multiply the radius cubed by the fraction value of $\frac{4}{3}$. The last thing we do is multiply by pi because sometimes we want to leave it in terms of pi. If we don't want to leave it in terms of pi, we multiply by the approximation of pi which is 3.14. Finally, don't forget the unit for the answer.

32

Lesson 8.5

Answer the following questions either using $\pi \approx 3.14$ or giving your answer in terms of π. Round your answer to the nearest hundredth where necessary.



1. Find the volume of a cylinder with a radius of 3 in and a height of 10 in.

2. Find the volume of a cylinder with a radius of 10 mm and a height of 2 mm.

3. Find the volume of a cylinder with a radius of 5 cm and a height of 15 cm.

4. Find the volume of a cylinder with a diameter of 22 m and a height of 5 m.

5. Find the volume of a cylinder with a diameter of 4 ft and a height of 1 ft.

6. Find the volume of a cylinder with a radius of 9 in and a height of 9 in.

7. Find the volume of a can of green beans with a radius of 3 cm and a height of 8 cm.

8. Find the volume of a cylindrical can of oatmeal with a radius of 8 cm and a height of 45 cm.

9. Find the volume of a cylindrical water bottle with a diameter of 4 cm and a height of 30 cm.

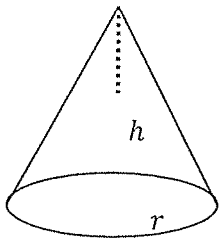
10. Find the volume of a can of Pepsi with a diameter of 2 in and a height of 3.5 in.

11. Find the volume of a water pipe with a radius of 0.75 ft and a length of 16 ft.

12. Find the volume of a straw used for drinking with a radius of 2 mm and a height of 170 mm.

33

13. Find the volume of a cone with a radius of 3 in and a height of 10 in.



14. Find the volume of a cone with a radius of 10 mm and a height of 3 mm.

15. Find the volume of a cone with a radius of 5 cm and a height of 15 cm.

16. Find the volume of a cone with a radius of 12 m and a height of 5 m.

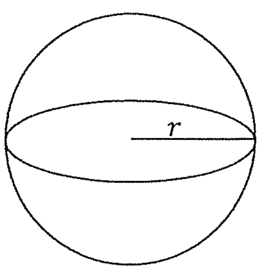
17. Find the volume of a cone with a diameter of 4 ft and a height of 9 ft.

18. Find the volume of a cone with a diameter of 18 in and a height of 9 in.

19. Find the volume of a waffle cone for ice cream with a radius of 4 cm and a height of 12 cm.

20. Find the volume of a cone birthday hat with a radius of 2 in and a height of 9 in.

21. Find the volume of a funnel with a diameter of 10 cm and a height of 9 cm.



22. Find the volume of a sphere with a diameter of 6 in.

23. Find the volume of a sphere with a diameter of 18 mm.

24. Find the volume of a sphere with a radius of 6 cm.

25. Find the volume of a sphere with a radius of 12 m.

26. Find the volume of a sphere with a radius of 2 ft.

27. Find the volume of a sphere with a radius of 5 in.

28. Find the volume of a mini basketball with a radius of 3.5 in.

29. Find the volume of the Earth with a diameter of approximately 12,756 km.

30. Find the volume of the moon with a diameter of approximately 3475 km.

31. Find the volume of a gumball with a radius of 3 mm.

34

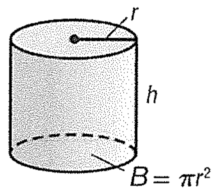
NAME ______________________ DATE ______________________ PERIOD __________

Reteach

Volume of Cylinders

As with prisms, the area of the base of a **cylinder** tells the number of cubic units in one layer. The height tells how many layers there are in the cylinder. The volume V of a cylinder with radius r is the area of the base B times the height h.

$V = Bh$, where $B = \pi r^2$, or $V = \pi r^2 h$

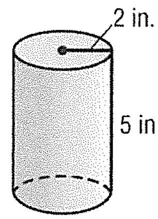


Example

Find the volume of the cylinder. Round to the nearest tenth.

$V \approx \pi r^2 h$	Volume of a cylinder
$V \approx \pi(2)^2(5)$	Replace r with 2 and h with 5.
$V \approx 62.8318$	Use a calculator

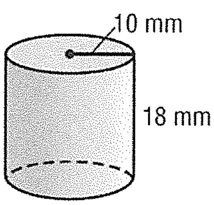
The volume is about 62.8 cubic inches.



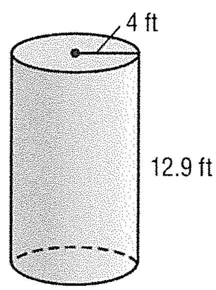
Exercises

Find the volume of each cylinder. Round to the nearest tent

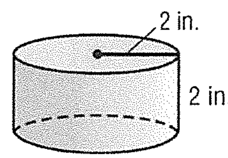
1.



2.



3.



4. radius = 9.5 yd
height = 2.2 yd

5. diameter = 6 cm
height = 11 cm

6. diameter = 3.4 m
height = 1.25 m

35

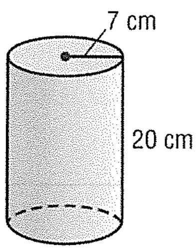
NAME ______________________________ DATE ____________________ PERIOD ________

Skills Practice

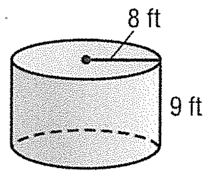
Volume of Cylinders

Find the volume of each cylinder. Round to the nearest tenth.

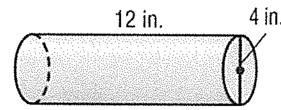
1.



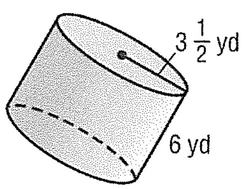
2.



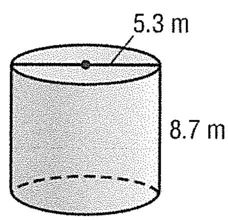
3.



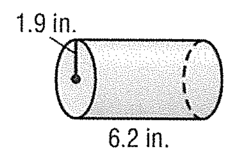
4.



5.



6.



7. radius = 8.8 cm
height = 4.7 cm

8. radius = 4 ft
height = $2\frac{1}{2}$ ft

9. diameter = 10 mm
height = 4 mm

10. diameter = 7.1 in.
height = 1 in.

11. diameter = 12 ft
height = 18 ft

12. diameter = $3\frac{1}{2}$ in.
height = 5 in.

36

NAME ______________________ DATE ______________ PERIOD ________

Problem-Solving Practice

Volume of Cylinders

1. **WATER STORAGE** A cylindrical water tank has a diameter of 5.3 meters and a height of 9 meters. What is the maximum volume that the water tank can hold? Round to the nearest tenth.

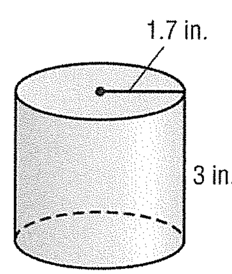
2. **PACKAGING** A can of corn has a diameter of 6.6 centimeters and a height of 9.9 centimeters. How much corn can the can hold? Round to the nearest tenth.

3. **CONTAINERS** Felisa wants to determine the maximum capacity of a cylindrical bucket that has a radius of 6 inches and a height of 12 inches. What is the capacity of Felisa's bucket? Round to the nearest tenth.

4. **GLASS** Antoine is designing a new, cylindrical drinking glass. If the glass has a diameter of 8 centimeters and a height of 12.8 centimeters, what is its volume? Round to the nearest tenth.

5. **PAINT** A can of paint is 15 centimeters high and has a diameter of 13.6 cm. What is the volume of the can? Round to the nearest tenth.

6. **SPICES** A spice manufacturer uses a cylindrical dispenser like the one shown. Find the volume of the dispenser to the nearest tenth.



NAME ______________________ DATE ______________ PERIOD ________

Reteach

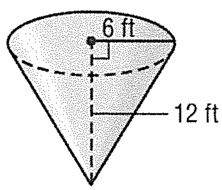
Volume of Cones

A **cone** is a three-dimensional shape with one circular base.

The volume V of a cone with radius r is one third the area of the base B times the height h.
$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

Example

Find the volume of the cone. Round to the nearest tenth.



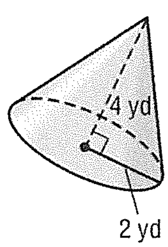
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$V = \frac{1}{3}(\pi \cdot 6^2 \cdot 12)$	$r = 6$ and $h = 12$
$V \approx 452.4$	Simplify.

The volume is about 452.4 cubic feet.

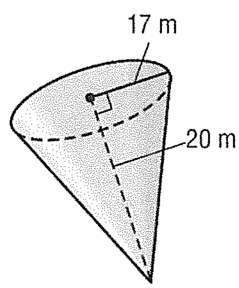
Exercises

Find the volume of each cone. Round to the nearest tenth.

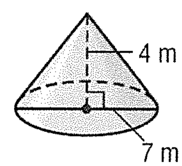
1.



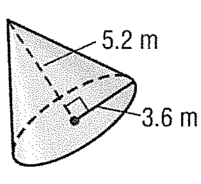
2.



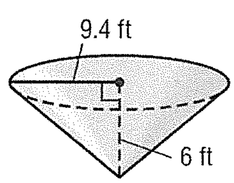
3.



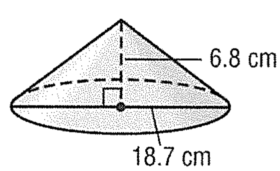
4.



5.



6.



38

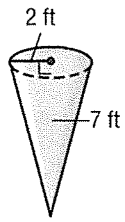
NAME ____________________ DATE ____________ PERIOD ________

Skills Practice

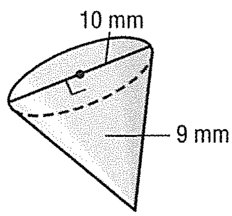
Volume of Cones

Find the volume of each cone. Round to the nearest tenth.

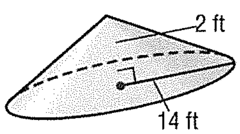
1.



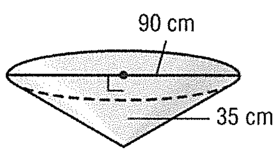
2.



3.



4.



5. diameter: 10 centimeters; height: 14 centimeters

6. radius: 8.7 feet; height: 16 feet

7. height: 34 centimeters; diameter: 6 centimeters

8. **FUNNEL** A funnel is in the shape of a cone. The radius is 2 inches and the height is 4.6 inches. Find the volume of the funnel. Round to the nearest tenth.

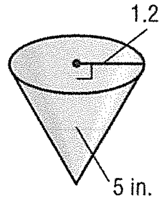
39

NAME ______________________ DATE ____________ PERIOD ________

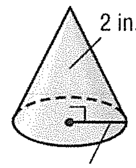
Problem-Solving Practice

Volume of Cones

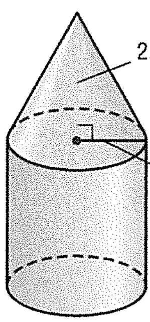
1. DESSERT Find the volume of the ice cream cone shown below. Round to the nearest tenth.



2. SALT Lecretia uses a small funnel as shown below to fill her salt shaker. Find the volume of the funnel. Round to the nearest tenth.



3. ENTRYWAY The top of the stone posts at the entry to an estate are in the shape of a cone as shown below. Find the volume of stone needed to make the top of the post. Round to the nearest tenth.



4. PAPERWEIGHT Marta bought a paperweight in the shape of a cone. The radius was 10 centimeters and the height 9 centimeters. Find the volume. Round to the nearest tenth.

5. LAMPSHADE A lampshade is in the shape of a cone. The diameter is 5 inches and the height 6.5 inches. Find the volume. Round to the nearest tenth.

6. CANDY A piece of candy is in the shape of a cone. The height of the candy is 2 centimeters and the diameter is 1 centimeter. Find the volume. Round to the nearest tenth.

NAME ______________________ DATE ____________ PERIOD ________

Reteach

Volume of Spheres

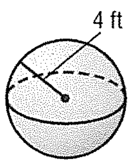
A **sphere** is a set of all points in space that are a given distance from a given point.

The volume V of a sphere with radius r is four thirds the product of π and the cube of the radius r.

$V = \frac{4}{3}\pi r^3$.

Example

Find the volume of the sphere. Round to the nearest tenth.



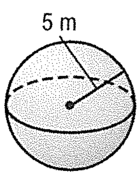
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$V = \frac{4}{3}(\pi \cdot 4^3)$	$r = 4$
$V \approx 268.1$	Simplify. Use a calculator.

The volume is about 268.1 cubic feet.

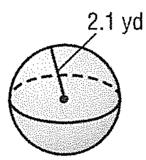
Exercises

Find the volume of each sphere. Round to the nearest tenth.

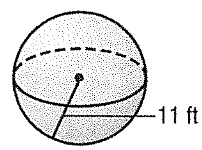
1.



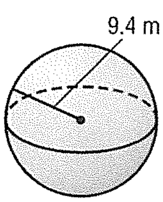
2.



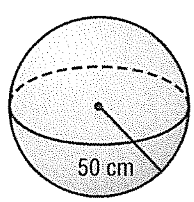
3.



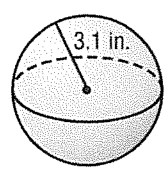
4.



5.



6.



41

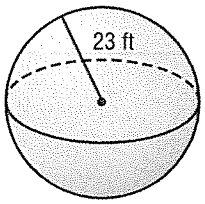
NAME ______________________ DATE ____________ PERIOD ________

Skills Practice

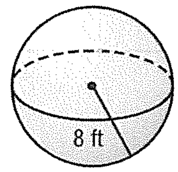
Volume of Spheres

Find the volume of each sphere. Round to the nearest tenth.

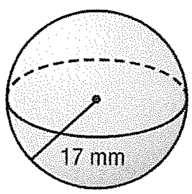
1.



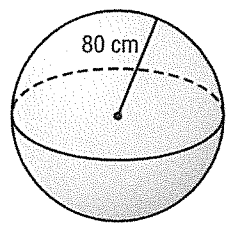
2.



3.

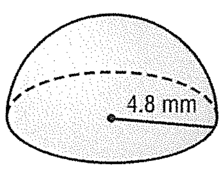


4.

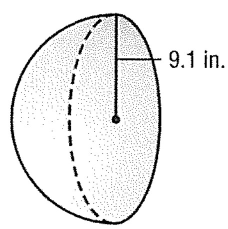


Find the volume of each hemisphere. Round to the nearest tenth.

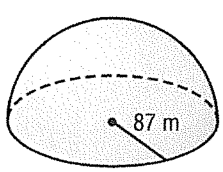
5.



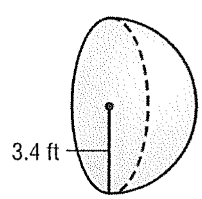
6.



7.



8.



42

NAME ______________________ DATE ____________ PERIOD ________

Problem-Solving Practice

Volume of Spheres

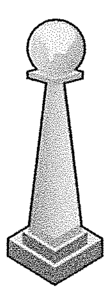
1. **DESSERT** A scoop of ice cream is in the shape of a sphere. The diameter of the scoop of ice cream is 2.5 inches. Find the volume of the ice cream. Round to the nearest tenth.

2. **TOYS** A playground ball has a radius of 7.5 inches. Find the volume of the ball. Round to the nearest tenth.

3. **GLOBE** A globe has a diameter of 14 inches. Find the volume of the globe. Round to the nearest tenth.

4. **JEWELRY** Jackie is using spherical beads to create a border on a picture frame. Each bead has a diameter of 1.5 millimeters. Find the volume of each bead. Round to the nearest tenth.

5. **DECORATION** A glass ball is used to decorate a garden. The radius of the ball is 25 centimeters. Find the volume. Round to the nearest tenth.



6. **BALLOONS** Mrs. McCullough is purchasing balloons for a party. Each spherical balloon is inflated with helium. How much helium is in the balloon if the balloon has a radius of 9 centimeters? Round to the nearest tenth.

8.6 Solving for a Missing Dimension

Given the volume of a shape, we can solve for a missing dimension such as the height or radius. It should come as no surprise that to isolate the variable in the equation, we will use inverse operations.

Solving for the Height

Let's start with a cylinder with a volume of approximately 314 in^3 and a radius of 5 $in.$ Since we know the formula for the volume of the cylinder, we can plug in and work backwards.

$$V = \pi r^2 h$$

Substitute what we know

$$314 = 3.14 * (5)^2 h$$

$$314 = 3.14 * 25 * h$$

Simplify

$$314 = 78.5h$$

$$\frac{314}{78.5} = \frac{78.5h}{78.5}$$

Solve

$$4 = h$$

So the height of the cylinder is 4 $in.$ Similarly, we can solve for the height if we know the volume of a cone. The difference is only the fraction. In most cases it will be easier to eliminate the fraction at the beginning. For example, consider a cone with a volume of 37.68 in^3 and a radius of 2 $in.$ Follow the same process as above to solve for the height.

$$V = \frac{1}{3}\pi r^2 h$$

Substitute what we know

$$37.68 = \frac{1}{3} * 3.14 * (2)^2 h$$

$$3 * 37.68 = 3 * \frac{1}{3} * 3.14 * (2)^2 h$$

Multiply by 3 to eliminate the fraction.

$$113.04 = 3.14 * 4 * h$$

$$113.04 = 12.56h$$

$$\frac{113.04}{12.56} = \frac{12.56h}{12.56}$$

Solve

$$9 = h$$

44

Solving for the Radius

When solving for the radius, we'll have to think back to our knowledge of square and cube roots. Since the radius is either squared or cubed in the volume formulas, we will need to apply either the square root or cube root as one of our inverse operations.

Let's look at a sphere with a volume of $904.32\, m^3$. We know the formula for volume, so we will substitute, simplify and solve.

$$V = \frac{4}{3}\pi r^3$$

$$904.32 = \frac{4}{3} * 3.14 * r^3$$

Substitute what we know

$$\frac{3}{4} * 904.32 = \frac{3}{4} * \frac{4}{3} * 3.14 * r^3$$

Multiply by $\frac{3}{4}$ to eliminate the fraction.

$$678.24 = 3.14 * r^3$$

$$\frac{678.24}{3.14} = \frac{3.14 * r^3}{3.14}$$

$$216 = r^3$$

$$\sqrt[3]{216} = \sqrt[3]{r^3}$$

Inverse Operations

$$6 = r$$

In this case the radius was $6\ m$, and one of the steps necessary in solving this was using the cube root. If we were solving for the radius in a cylinder or cone, we would need the square root.

45

Lesson 8.6

Answer the following questions using $\pi \approx 3.14$. Round your answer to the nearest hundredth where necessary.

1. Find the height of a cylinder with a volume of $30\ in^3$ and a radius of $1\ in$.

2. Find the height of a cylinder with a volume of $100\ cm^3$ and a radius of $2\ cm$.

3. Find the height of a cylinder with a volume of $720\pi\ ft^3$ and a radius of $6\ ft$.

4. Find the height of a cylinder with a volume of $1215\pi\ mm^3$ and a radius of $9\ mm$.

5. Find the radius of a cylinder with a volume of $950\ in^3$ and a height of $10\ in$.

6. Find the radius of a cylinder with a volume of $208\ cm^3$ and a height of $4\ cm$.

7. Find the radius of a cylinder with a volume of $108\pi\ ft^3$ and a height of $12\ ft$.

8. Find the radius of a cylinder with a volume $686\ mm^3$ and a height of $14\ mm$.

9. Find the height of a cone with a volume of $150\ in^3$ and a radius of $10\ in$.

10. Find the height of a cone with a volume of $21\ ft^3$ and a radius of $4\ ft$.

11. Find the radius of a cone with a volume of $175\ cm^3$ and a height of $21\ cm$.

12. Find the radius of a cone with a volume of $196\pi\ mm^3$ and a height of $12\ mm$.

13. Find the radius of a sphere with volume $\approx 113.04\ in^3$.

14. Find the radius of a sphere with volume $\approx 904.32\ cm^3$.

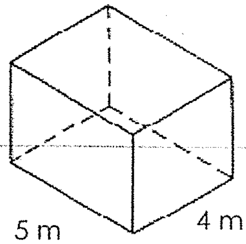
15. Find the radius of a sphere with volume $\approx 3052.08\ m^3$.

16. Find the radius of a sphere with volume $\approx 4.18\bar{6}\ ft^3$.

46

VOLUME OF A PRISM OR CYLINDER

RECTANGULAR PRISM



2 m

5 m 4 m

V = Bh

L x w

5 x 4 x 2

20 x 2

40 m³

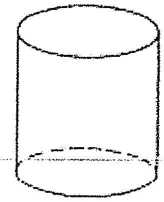
The base is a rectangle, so under "Big B" write "l x w" for the area of a rectangle.

V = Bh

Area of the base

Choose an area formula to plug into "Big B"

CYLINDER



5 m

d = 4 m

Make sure you find the radius first!

If the diameter is 4 m, then the radius is 2 m!

V = Bh

π x r²

3.14 x 2² x 5

3.14 x 2 x 2 x 5

6.28 x 2 x 5

12.56 x 5

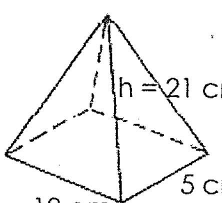
62.8 m³

The base is a circle, so under "Big B" write "π x r²" for the area of a circle.

VOLUME OF A PYRAMID OR CONE

RECTANGULAR PYRAMID

$V = \frac{1}{3}Bh$



h = 21 cm

18 cm 5 cm

L x W x h

18 x 5 x 21

90 x 21

1890

Don't forget 1/3 means divide by 3

$1890 \div 3 = 630$

630 cm³

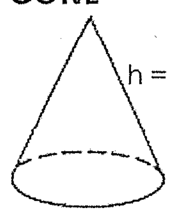
$V = \frac{1}{3}Bh$

Area of the base

$\frac{1}{3}$ Means DIVIDE by 3

CONE

$V = \frac{1}{3}Bh$



h = 20 cm

r = 10 cm

π x r² x h

3.14 x 10² x 20

3.14 x 2000

6280

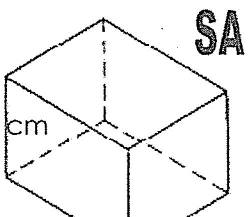
Don't forget 1/3 means divide by 3

$6280 \div 3 = 2093$

2093 cm³

SURFACE AREA OF PRISMS

SA = 2(lw) + 2(wh) + 2(lh)



5 cm

12 cm 6 cm

2(lw)	2(wh)	2(lh)
2 x 12 x 6	2 x 6 x 5	2 x 12 x 5
24 x 6	12 x 5	24 x 5
144	60	120

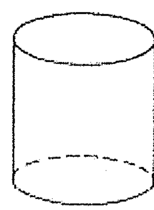
144 + 60 + 120

204 + 120

324 cm²

SURFACE AREA OF CYLINDERS

SA = 2πr² + 2πrh



h = 13 in

d = 14 in

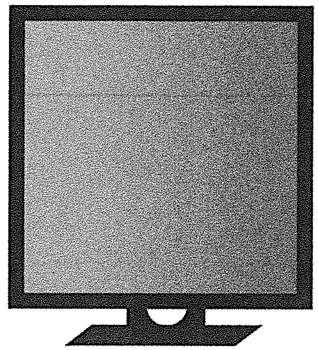
Don't forget if you're given the diameter to divide by 2 for the radius!

2πr²	2πrh
2 x π x r²	2 x π x r x h
2 x 3.14 x 7²	2 x 3.14 x 7 x 13
2 x 3.14 x 49	2 x 3.14 x 91
6.28 x 49	6.28 x 91
307.72	571.48

307.72 + 571.48

879.2 in²

Comparing TVs



Vehicle Viewing, Inc. and Traveling TV Co. have each designed new small HD flat-screen square-shaped televisions to be used in automobiles. They feel that these will be very popular because of the desire for parents to entertain their children during the many hours spent in traffic each day. Television size is measured along the diagonal. The diagonal in the Vehicle Viewing, Inc. television is $\sqrt{75}$ inches and the Traveling TV Co. television has a diagonal of 8.5 inches. You are the owner of a packaging company and both Vehicle Viewing, Inc. and Traveling TV Co. have hired you to package their new televisions.

1. Who has the larger television? How do you know?

2. To protect the screen, you need to place a protective foam sheet between the screen and the box. Find the area of each television screen so that you know how much sheeting you will need to order. Verify your results.

3. In order to prevent breakage, you will need to put some foam ribbon around the sides of the television. Exactly how much foam ribbon will need to be used for each television? Justify your answer.

TACO CART

Name:______________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.