UNIT 3: FUNCTIONS

Worth County Middle School

2015-2016

Unit 3: Functions

BIG IDEAS

- A function is a specific type of relationship in which each input has a unique output.
- A function can be represented in an input-output table, graphically (using ordered pairs that consist of the input and the output of the function in the form (input, output), and with an algebraic rule

STANDARDS FOR MATHEMATICAL CONTENT

Define, evaluate, and compare functions.
MGSE8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
MGSE8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, give a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

ESSENTIAL QUESTIONS

- What is a function?
- What are the characteristics of a function?
- How do you determine if relations are functions?
- How is a function different from a relation?
- Why is it important to know which variable is the independent variable?
- How can a function be recognized in any form?
- What is the best way to represent a function?
- How do you represent relations and functions using tables, graphs, words, and algebraic equations?
- What strategies can I use to identify patterns?
- How does looking at patterns relate to functions?
- How are sets of numbers related to each other?
- How can you use functions to model real-world situations?
- How can graphs and equations of functions help us to interpret real-world problems?

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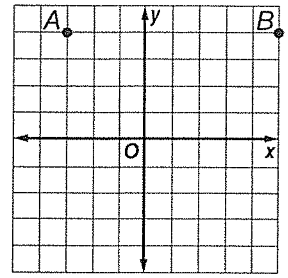
Reteach

Relations

Example 1

Name the ordered pair for point *A*.



- Start at the origin.
- Move left on the x-axis to find the x-coordinate of point A, which is -3.
- Move up the y-axis to find the y-coordinate, which is 4.

So, the ordered pair for point A is $(-3, 4)$.

Example 2

Graph point *B* at (5, 4).

- Use the coordinate plane shown above. Start at the origin and move 5 units to the right. Then move up 4 units.
- Draw a dot and label it $B(5, 4)$.

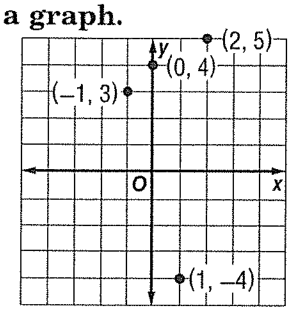
Example 3

Express the relation {(2, 5), (−1, 3), (0, 4), (1, −4)} as a table and a graph. Then state the domain and range.

The domain is $\{-1, 0, 1, 2\}$.

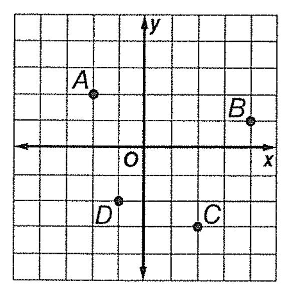
The range is $\{-4, 3, 4, 5\}$.

x	y
2	5
−1	3
0	4
1	−4



Exercises

Name the ordered pair for each point.

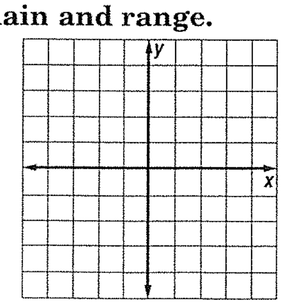


1. *A*
2. *B*
3. *C*
4. *D*

Express the relation as a table and a graph. Then state the domain and range.

5. $\{(-3, 1), (2, 4), (-1, 0), (4, -4)\}$

x	y



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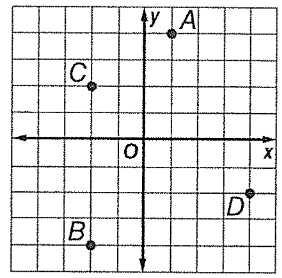
Skills Practice

Relations

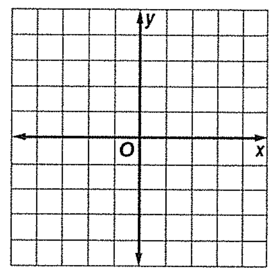
Name the ordered pair for each point.



1. *A* **2.** *B*

3. *C* **4.** *D*

Graph each ordered pair on a coordinate plane.



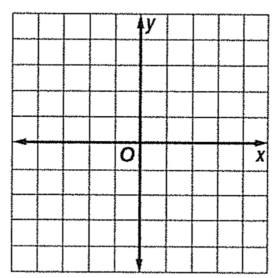
5. (3, 3) **6.** (1, −1)

7. (−4, 2) **8.** (−4, −3)

Express each relation as a table and a graph. Then state the domain and range.

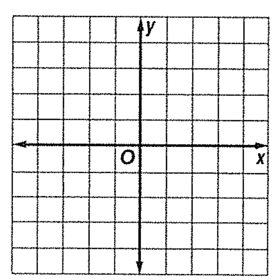
9. {(4, −2), (−1, 1), (2, −3), (3, 0)}

x	y



10. {(3, 4), (1, −2), (4, −1), (2, 2)}

x	y



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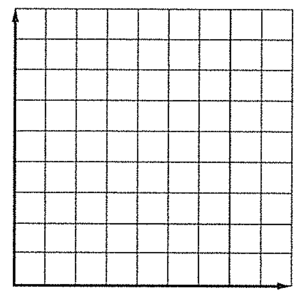
Problem-Solving Practice

Relations

1. MONEY The Happy Place charges \$30 per hour for parties. Make a table of ordered pairs in which the x-coordinate represents the hours and the y-coordinate represents the total cost for 2, 3, 4, and 5 hours.

x	y

2. Graph the ordered pairs from Exercise 1 and state the domain and range.



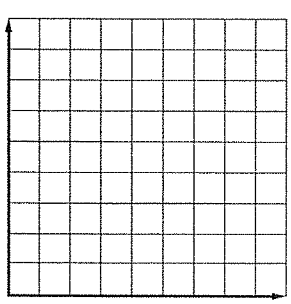
3. CAR RENTALS The ABC Car Rental Company charges a flat rate \$58 per day. Make a table of ordered pairs in which the x-coordinate represents the number of days and the y-coordinate represents the total cost for 1, 3, 5, and 7 days.

x	y

4. PRODUCE A company that sells produce fills 350 boxes of squash per day. Make a table of ordered pairs in which the x-coordinate represents the number of days and the y-coordinate represents the number of boxes filled in 1, 2, 3, and 4 days.

x	y

5. Graph the ordered pairs from Exercise 4 and state the domain and range.



6. BABIES Shaqueem's baby brother drinks 4 ounces of formula every 3 hours. Make a table of ordered pairs in which the x-coordinate represents the number of hours and the y-coordinate represents the total number of ounces in 3, 6, 9, and 12 hours.

x	y

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1.1 Defining Functions

Functions govern many interactions in our society today. Whether buying a cup of coffee at the local coffee shop or playing a video game, we are using a function in some fashion.

Definition of a Function

A **function** is a rule or relationship between two quantities, often referred to as the input and output, such that for every input there is exactly one output. If we input a specific value into the function, we get a specific output as an answer. We won't get the possibility of two answers or else it wouldn't be a function. The most common example of a function is an equation such as:

$$y = 2x + 3$$

In this case, x is the input and y is the output. If we substitute a value for x, say $x = 3$, then we will get an answer for y, namely $y = 9$, as the output. Notice that every time we input $x = 3$ you will get the output of $y = 9$. Since we always get only a single output for any value we input, this is a true function.

An example of an equation that is not a function would be $y^2 = x$. Notice that if we input $x = 4$, then $y = 2$ could be the output or $y = -2$ could be the output. Therefore this is not a true function unless we make the function $y = \sqrt{x}$ where we take only the principal (or positive) square root.

Function Notation

Functions represented by equations have a different notation. We are used to an equation looking like this: $y = 2x + 3$. However, from this point forward function equations will be written as follows:

$$f(x) = 2x + 3$$

We would say that f is a function of x such that if you input x, you will output $2x + 3$. The advantage of this notation is that we clearly know what our input and output are. For example, we might use the function $h(t) = -16t^2 + 48t$ to represent the height of ball thrown in the air over time. We would say that the height of the ball, h, is a function of the time since you have thrown it in the air, t.

Evaluating Functions

Evaluating a function means to figure out what the output is when given a specific input. Let's look at the following function that shows the total cost at an amusement park, c, depending on the number of tickets bought, t, to ride the rides.

$$c(t) = 2t + 3$$

We can evaluate this function for $t = 5$, or $c(5)$, by substituting into the equation as follows:

$$c(5) = 2(5) + 3 = 10 + 3 = 13$$

So our output is $c(5) = 13$ meaning it the cost to buy 5 tickets is $13. Let's evaluate the same function for 10 tickets by looking for $c(10)$.

$$c(10) = 2(10) + 3 = 20 + 3 = 23$$

Our output this time is $c(10) = 23$ meaning the cost to buy 10 tickets is $23.

Domain of a Function

The set of all possible inputs is called the **domain** of a function. For $f(x) = 2x + 3$ the domain is all real numbers. Any number we want could be input into the function as x. One way we can write this is in set notation as follows:

$$D: (-\infty, \infty)$$

This means that the domain of the function is any number between negative infinity and infinity. So we can input any number we want for x. However, in the context of the carnival as described above, it would only make sense to think about the domain $D: [0, \infty)$ since we wouldn't buy negative tickets.

Notice the bracket [instead of the parentheses. The bracket means that it can equal that number. When we use the parentheses, we mean it cannot actually equal the number. So if we can input zero and above (greater than or equal to zero) we use the domain $D: [0, \infty)$, but if we can only input numbers strictly greater than zero, we use the domain $D: (0, \infty)$.

Some functions have limited domains or ranges. For example, in the function $f(x) = \sqrt{x + 5}$ we can only input $x \geq -5$ because we can't take the square root of a negative. This means the domain would be written as $D: [-5, \infty)$.

Another example of a limited domain is the function $f(x) = \frac{100}{x}$ which has a domain of any number except 0 (since we can't divide by 0). We might write this out by saying the domain is $D: x \neq 0$. Note that this is not in set notation, but rather written as an inequality. There is nothing wrong with using the most efficient method to communicate the domain.

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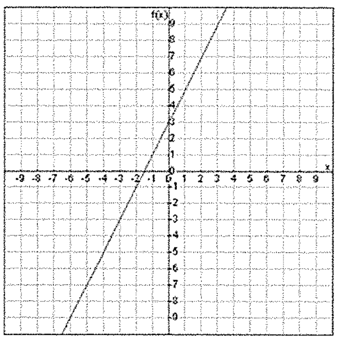
Range of a Function

Similar to the domain, all of the outputs we could possibly get are called the **range**. More precisely, the range is the set of all possible outputs for a function. The range for the function $f(x) = 2x + 3$ is all real numbers. This means the output, or $f(x)$ value, could be any number and would write the range of this function as $R: (-\infty, \infty)$.

Consider the function $f(x) = x^2 - 2$. Notice that no matter what value we plug in for x, we will always output a number greater than or equal to -2. Therefore we would write this range as $R: [-2, \infty)$.

It is often easier to see the domain and range from the graph of a function. Let's consider a few examples.

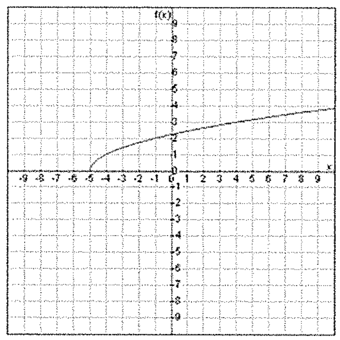
$f(x) = 2x + 3$



Domain: Any x because the graph continues forever both left and right. $D: (-\infty, \infty)$

Range: Any output because the graph continues forever both up and down. $R: (-\infty, \infty)$

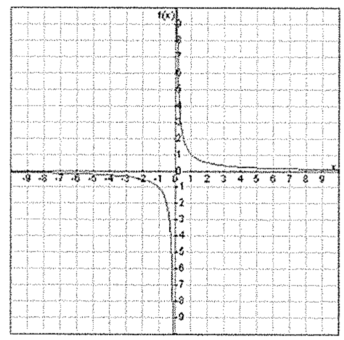
$f(x) = \sqrt{x+5}$



Domain: The inputs must be greater than or equal to -5 because the graph continues forever to the right starting at -5. $D: [-5, \infty)$

Range: All outputs will be greater than or equal to 0 because the graph continues forever up from the height of 0. $R: [0, \infty)$

$f(x) = \frac{1}{x}$



Domain: $x \neq 0$ because the graph continues forever to the left and right but never touches $x = 0$. $D: x \neq 0$

Range: The graph continues forever up and down but never has a height of $y = 0$. $R: f(x) \neq 0$

Is it a Function?

So how exactly can we tell if something is a function? The definition is that every input has only one output. This means that not only equations can be functions, but graphs, tables, and words can be functions. For example consider the following examples of potential functions.

Example 1
Input: A person's identity Output: Their social security number

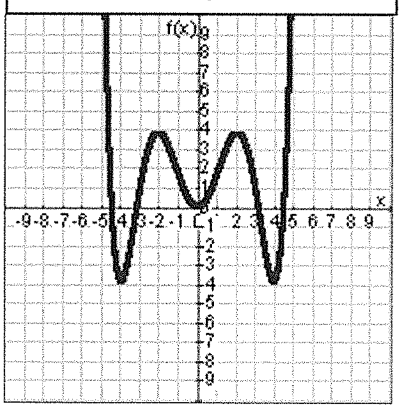
Example 2
Input: A person's age Output: Their weekly income

In this first example, if we input someone's identity, say your math teacher, will we output only one social security number? Yes. One person never has two social security numbers. So example one is a true function. The domain of the function would be any citizen of the USA (since social security is our thing) and the range would be all the social security numbers.

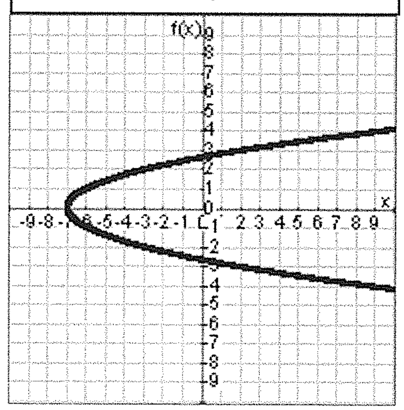
In the second example, if we input someone's age, say 33 years old, will we output only one weekly income? No. One thirty-three year old could make \$200 per week and another could make \$1,000 per week. That is two outputs for one input, so it is not a function. Since it's not a function, we don't have to worry about the domain and range.

Sometimes it's easier to see whether something is a function in graph form. Consider the following two graphs and decide if they are functions or not.

Example 3



Example 4



Here we can use what is called the vertical line test. Since the inputs are the x values, if for any x value at all we get multiple outputs (or y values on the graph), then it is not a function. Take a pencil and lay it down vertically (up and down) on the left side of Example 3. Now slowly push the pencil to the right. Is there any place where the pencil touches the graph more than once? No. That means every input has only one output, so it is a function. Any input looks fine and the graph is always higher than -4. Therefore the domain is $D: (-\infty, \infty)$ and the range is approximately $R: (-4, \infty)$ since it looks like it doesn't quite touch the height of -4.

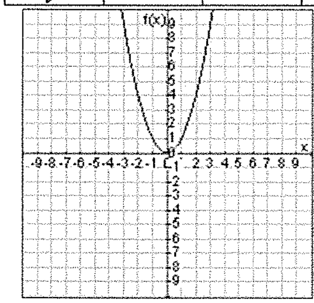
If you do the same thing on Example 4, the pencil will hit the graph in two places starting to the right of $x = -7$. One particular example is look at $x = -3$. In that case we have outputs of both $y = 2$ and $y = -2$. With two outputs, the vertical line hitting twice, this cannot be a function.

Lesson 1.1

Determine if each of the following is a true function or not and explain how you know. If it is a true function, give the domain and range.

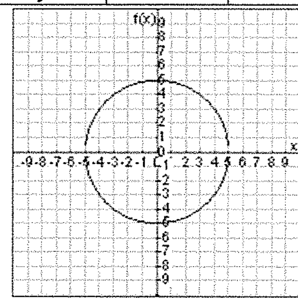
1. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4



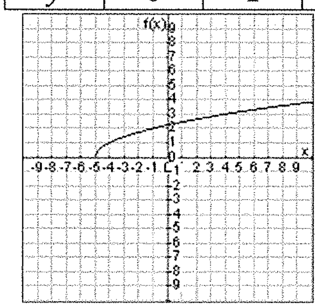
2. $x^2 + y^2 = 25$

x	-4	-3	0	3	4
y	± 3	± 4	± 5	± 4	± 3



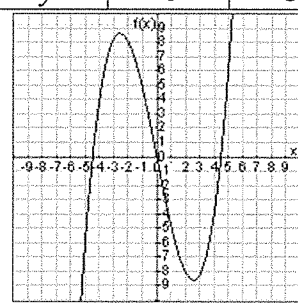
3. $y = \sqrt{x + 5}$

x	-5	-4	-1	4	11
y	0	1	2	3	4



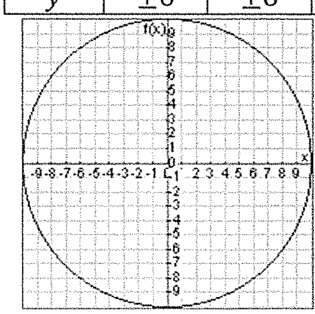
4. $y = \frac{1}{4}x^3 - 5x$

x	-4	-2	0	2	4
y	4	8	0	-8	-4



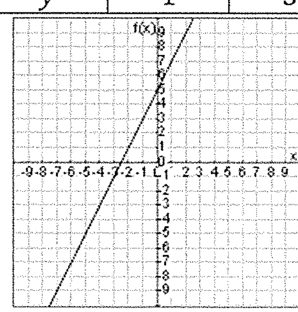
5. $x^2 + y^2 = 100$

x	-8	-6	0	6	8
y	± 6	± 8	± 10	± 8	± 6



6. $y = 2x + 5$

x	-2	-1	0	1	2
y	1	3	5	7	9



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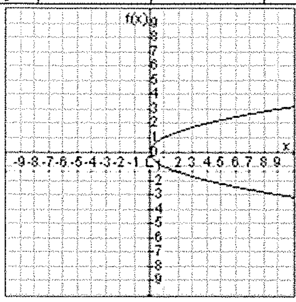
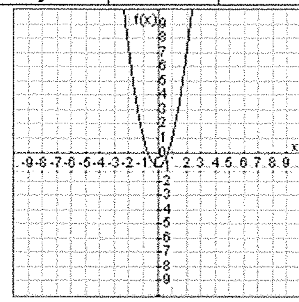
7. $x = y^2$

x	0	1	4	9	25
y	0	±1	±2	±3	±5



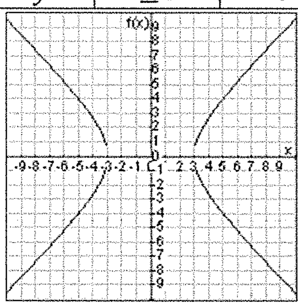
8. $y = 2x^2 - 1$

x	-2	-1	0	1	2
y	7	1	-1	1	7



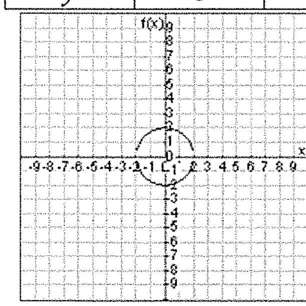
9. $x^2 - y^2 = 9$

x	-5	-3	3	5
y	±4	0	0	±4



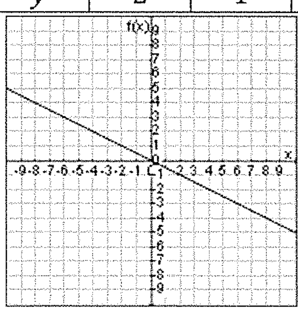
10. $\frac{x^2}{4} + \frac{y^2}{4} = 1$

x	-2	0	2
y	0	±4	0



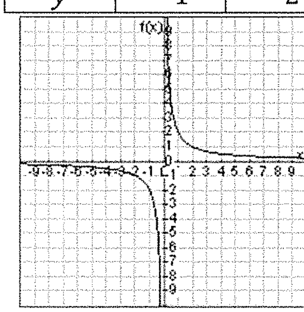
11. $y = -\frac{1}{2}x$

x	-4	-2	0	2	4
y	2	1	0	-1	-2



12. $y = \frac{2}{x}$

x	-2	-1	1	2
y	-1	-2	2	1



13. Explain how to determine whether or not an equation models a function.

14. Explain how to determine whether or not a table models a function.

Determine if the following descriptions of relationships represent true functions and explain why they do or why they do not. If it is a true function give a description of the domain and range.

15. Input: Time elapsed, Output: Distance run around the track.

16. Input: Store's name, Output: Number of letters in the name.

17. Input: Person's age, Output: Yearly salary.

18. Input: Amount of food eaten, Output: A dog's weight.

19. Input: Person's name/identity, Output: That person's birthday.

20. Input: Person's age, Output: Height.

21. Input: Name of a food, Output: Classification of that food (such as meat, dairy, grain, fruit, vegetable).

22. Input: Time studied for test, Output: Test score.

Determine if the following sequences represent true functions and explain why they do or why they do not. If it is a true function give a description of the apparent domain and range.

23. $0,1,2,3,4,5 \ldots$

24. $0, 1\ or\ 0, 1\ or\ 2, 3, 4, 5 \ldots$

25. $0,1,0,1,0,1 \ldots$

26. $1,1,1,1,1,1 \ldots$

27. $0\ or\ 1, 0\ or\ 1, 0\ or\ 1, 0\ or\ 1, 0\ or\ 1, 0\ or\ 1 \ldots$

28. $0,1,1,2,3,5 \ldots$

29. $1,2,4,8,16,32 \ldots$

30. $1,3,5,7,9,11 \ldots$

Evaluate the given functions at the given inputs.

$f(x) = x^2 + 2$ $\qquad$ $h(t) = \frac{1}{4}t - 3$ $\qquad$ $v(s) = s^3$

31. $f(-2)$ $\qquad$ 32. $f(10)$ $\qquad$ 33. $f(-3)$ $\qquad$ 34. $f(0)$

35. $h(-2)$ $\qquad$ 36. $h(-8)$ $\qquad$ 37. $h(4)$ $\qquad$ 38. $h(0)$

39. $v(2)$ $\qquad$ 40. $v(-2)$ $\qquad$ 41 $v(4)$ $\qquad$ 42. $v(1)$

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Reteach

Functions

A **function** is a relation in which each member of the domain (input value) is paired with exactly one member of the range (output value). You can organize the input, rule, and output of a function using a function table.

Example 1

Choose four values for x to make a function table for $f(x) = 2x + 4$. Then state the domain and range of the function.

Substitute each domain value x, into the function rule. Then simplify to find the range value.

$f(x) = 2x + 4$

$f(-1) = 2(-1) + 4$ or 2

$f(0) = 2(0) + 4$ or 4

$f(1) = 2(1) + 4$ or 6

$f(2) = 2(2) + 4$ or 8

The domain is {−1, 0, 1, 2}. The range is {2, 4, 6, 8}.

Input, x	Rule, $2x + 4$	Output, $f(x)$
−1	2(−1) + 4	2
0	2(0) + 4	4
1	2(1) + 4	6
2	2(2) + 4	8

Exercises

Find each function value.

1. $f(1)$ if $f(x) = x + 3$
2. $f(6)$ if $f(x) = 2x$
3. $f(4)$ if $f(x) = 5x - 4$
4. $f(9)$ if $f(x) = -3x + 10$
5. $f(-2)$ if $f(x) = 4x - 1$
6. $f(-5)$ if $f(x) = -2x + 8$

Choose four values for x to make a function table for each function. Then state the domain and range of the function.

7. $f(x) = x - 10$
8. $f(x) = 2x + 6$
9. $f(x) = 2 - 3x$

x	$x - 10$	$f(x)$

x	$2x + 6$	$f(x)$

x	$2 - 3x$	$f(x)$

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Skills Practice

Functions

Find each function value.

1. $f(2)$ if $f(x) = x + 4$

2. $f(9)$ if $f(x) = x - 8$

3. $f(3)$ if $f(x) = 2x + 2$

4. $f(6)$ if $f(x) = 2x - 5$

5. $f(-7)$ if $f(x) = 3x + 6$

6. $f(8)$ if $f(x) = 3x - 10$

7. $f(-5)$ if $f(x) = 4x + 2$

8. $f(-3)$ if $f(x) = -4x - 4$

9. $f(-4)$ if $f(x) = -5x - 3$

Choose four values for x to make a function table for each function. Then state the domain and range of the function.

10. $f(x) = x + 7$

x	$x + 7$	$f(x)$

11. $f(x) = x - 13$

x	$x - 13$	$f(x)$

12. $f(x) = 2x + 8$

x	$2x + 8$	$f(x)$

13. $f(x) = 2x - 3$

x	$2x - 3$	$f(x)$

14. $f(x) = 3x + 4$

x	$3x + 4$	$f(x)$

15. $f(x) = 7 - 3x$

x	$7 - 3x$	$f(x)$

16. $f(x) = 4x + 5$

x	$4x + 5$	$f(x)$

17. $f(x) = 1 - 4x$

x	$1 - 4x$	$f(x)$

18. $f(x) = 6x - 2$

x	$6x - 2$	$f(x)$

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Problem-Solving Practice

Functions

1. **JOBS** Strom works as a valet at the Westside Mall. He makes \$48 per day plus \$1 for each car that he parks. The total amount that Strom earns in one day can be found using the function $f(x) = x + 48$, where x represents the number of cars that Strom parked. Make a function table to show the total amount that Strom makes in one day if he parks 25 cars, 30 cars, 35 cars, and 40 cars.

x	$x + 48$	$f(x)$

2. **PLUMBING** Rico's Plumbing Service charges \$80 for a service call plus \$65 per hour for labor. The total charge can be found using the function $f(x) = 65x + 80$, where x represents the number of hours of labor. Make a function table to show the total amount that Rico's Plumbing Service charges if a job takes 1 hour, 2 hours, 3 hours, and 4 hours.

x	$65x + 80$	$f(x)$

3. **GEOMETRY** The perimeter of an equilateral triangle equals 3 times the length of one side. Write a function using two variables for this situation. Find the perimeter of an equilateral triangle with sides 18 inches.

4. **HEALTH CLUB** Courtney belongs to a health club that charges a monthly fee of \$20, plus \$85 to join. Write a function to represent her costs. How much has she paid after six months?

5. **LIBRARY FINES** The amount that Sunrise Library charges for an overdue book is \$0.25 per day plus a \$1 service charge. Write a function using two variables for this situation.

6. **LIBRARY FINES** Explain how to find the amount of the fine the library in Exercise 5 will charge for a book that is overdue by 12 days. Then find the amount.

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3.3 Linear and Non-Linear Functions

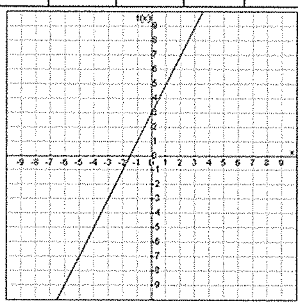
Most functions of cost are linear functions. If every cup of coffee costs $1.60 then two cups would cost $3.20, three cups would cost $4.80, and so forth. Let's explore what makes a function linear.

Definition of a Linear Function

Here are some examples of linear functions in equation, table, graph, and story form.

$c = 2t + 3$

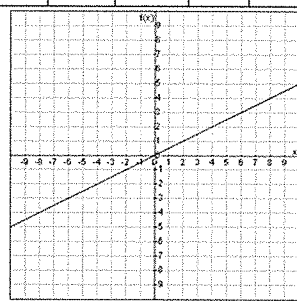
t	-2	-1	0	1	2
c	-1	1	3	5	7



A carnival charges $3 to enter and $2 per ticket for riding rides.

$s = \frac{1}{2}w$

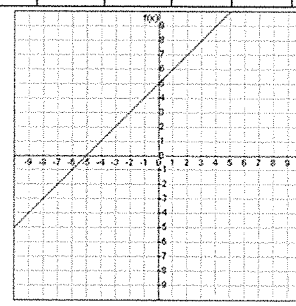
w	-2	-1	0	1	2
s	-1	$-.5$	0	$.5$	1



A company can make one sweater for every two pounds of wool it has.

$h = d + 5$

d	-2	-1	0	1	2
h	3	4	5	6	7



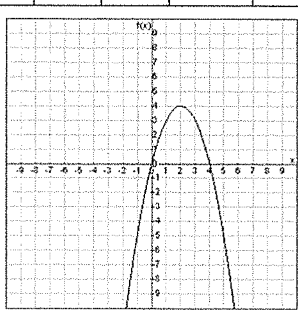
A plant today has a height of 5 mm and grows one mm each day.

What do you think makes these functions linear? Do you notice anything they have in common?

Here are some examples of **non-linear functions**. Can you define linear functions now?

$h = -(t-2)^2 + 4$

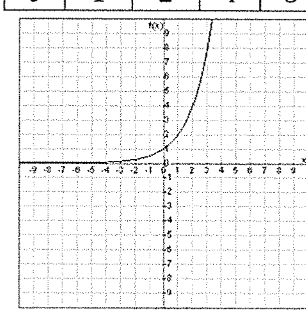
t	0	1	2	3	4
h	0	-5	-12	-21	-32



A rocket's height after t seconds is given by the above equation.

$c = 2^d$

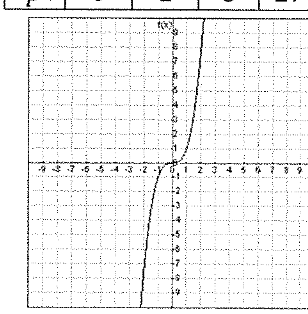
d	0	1	2	3	4
c	1	2	4	8	16



The number of cicadas double each day after the initial day one hatches.

$p = f^3$

f	0	1	2	3	4
p	0	1	8	27	64



Your dad pays you a number of pennies equal to how many flies you've killed cubed.

A **linear function** is a function that makes a straight line when graphed. Thus **non-linear** functions are any functions that are not linear. Graphing may be the quickest way to tell if a function is linear or non-linear, but we can also determine if a function is linear from its input/output table or equation.

This can be seen in the input/output table in that there is a constant difference in the dependent variable values. In other words, when the independent variable increases by one, the dependent variable will always increase or decrease the same amount. Let's look at our three linear tables again.

t	-2	-1	0	1	2
c	-1	1	3	5	7

w	-2	-1	0	1	2
s	-1	$-.5$	0	$.5$	1

d	-2	-1	0	1	2
h	3	4	5	6	7

Notice that the first table has a constant difference of two, meaning that as t increases by one, c increases by two every time. The second table has a constant difference of half, and the third table has a constant difference of one.

Alternately, notice that all three equations have the independent variable only to the first power. Meaning there is no exponent showing with the variable. That also means they are linear functions.

$c = 2t + 3$ $\qquad$ $s = \frac{1}{2}w$ $\qquad$ $h = d + 5$

Check the non-linear functions given on the previous page and see that they are not a straight line when graphed, have no constant difference, and have exponents in their equation.

In general, anything of the form $\boldsymbol{y = mx + b}$ is considered a **linear function** where m is the slope and b is called the y-intercept. Notice that the x has an unwritten exponent of one with it.

There are times when a linear function is not given in slope-intercept form. (That's what we call $y = mx + b$.) Sometimes a linear function is given in standard form which is $Ax + By = C$. However, since the exponent on the x variable is still a one, we can get it in slope intercept form. For example, consider the following:

$$2x + 3y = 6$$

$$2x + 3y - 2x = 6 - 2x$$

$$3y = -2x + 6$$

$$\frac{3y}{3} = \frac{-2x + 6}{3}$$

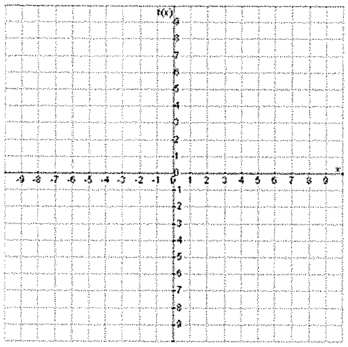
$$y = -\frac{2}{3}x + 2$$

Lesson 3.3

Determine whether the following functions are linear or non-linear and explain how you know. Blank x/y charts and coordinate planes have been given to graph the functions if that helps you.

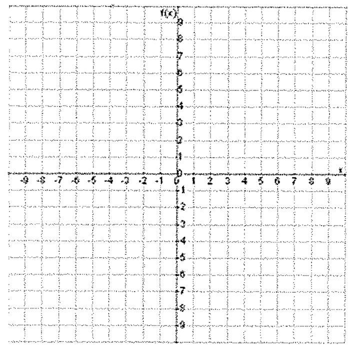
1. $y = x^2 - 2x$

x					
y					



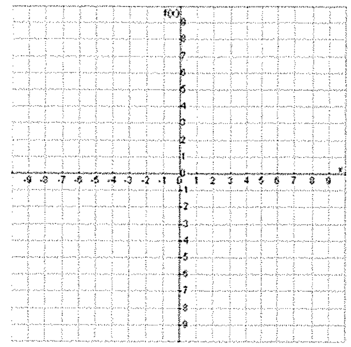
2. $y = \frac{1}{3}x - 2$

x					
y					



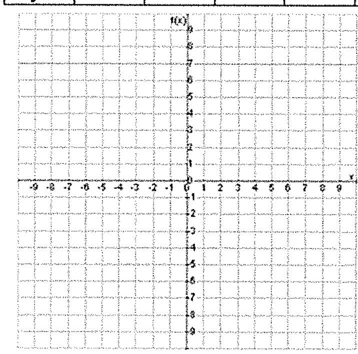
3. $y = -2x + 2$

x					
y					



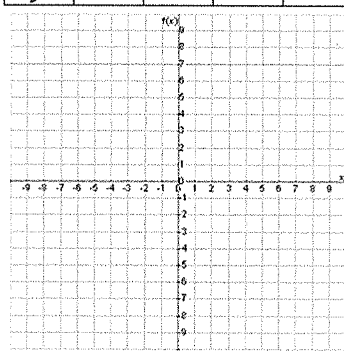
4. $y = x^2 + 3$

x					
y					



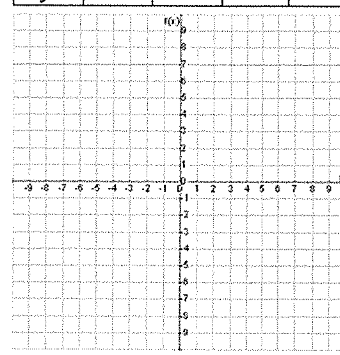
5. $5x + 3y = 0$

x					
y					



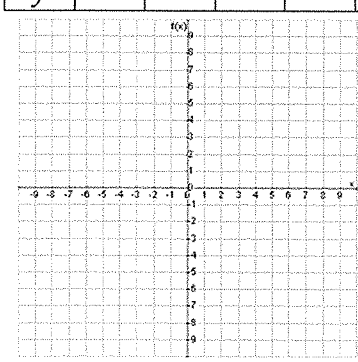
6. $y - 4x = -5$

x					
y					



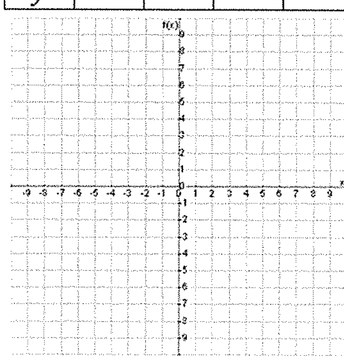
7. $y = \sqrt{x + 9}$

x					
y					



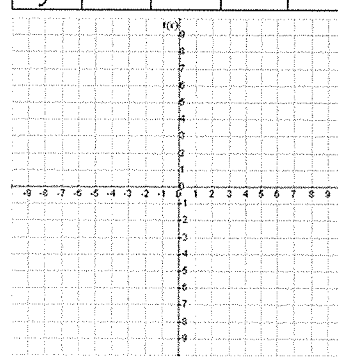
8. $y = 3^x - 2$

x					
y					



9. $y = x^3 - x^2$

x					
y					



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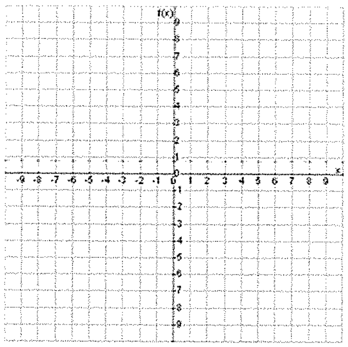
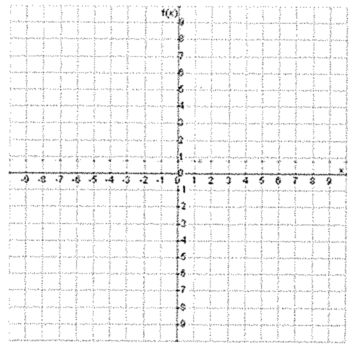
10. $y = x^3 - 7x$

x					
y					



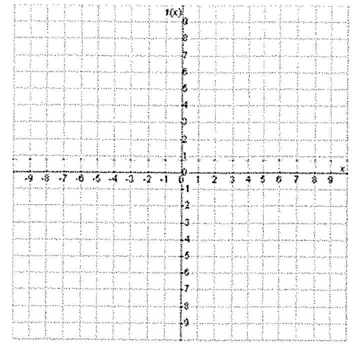
11. $y = 2^x + 3$

x					
y					



12. $y = \sqrt{x - 2}$

x					
y					

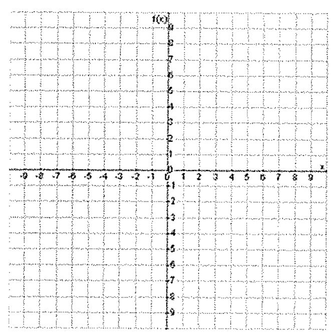


13. Give an example of a linear function in equation form.

14. Give an example of a linear function in table form.

x					
y					

15. Sketch an example of a linear function in graph form.

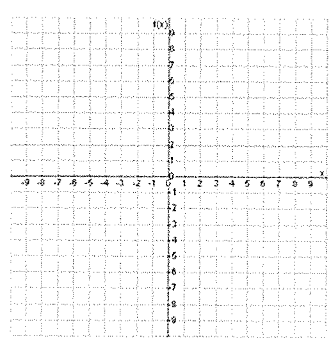


16. Give an example of a non-linear function in equation form.

17. Give an example of a non-linear function in table form.

x					
y					

18. Sketch an example of a non-linear function in graph form.



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Reteach

Linear and Nonlinear Functions

Linear functions represent constant rates of change. The rate of change for nonlinear functions is not constant. That is, the values do not increase or decrease at the same rate. You can use a table to determine if the rate of change is constant.

Example 1

Determine whether the table represents a *linear* or a *nonlinear* function. Explain.

+2 +2 +2

x	3	5	7	9
y	7	10	13	16

+3 +3 +3

As x increases by 2, y increases by 3. The rate of change is constant, so this function is linear.

Example 2

Determine whether the table represents a *linear* or a *nonlinear* function. Explain.

+1 +1 +1

x	1	2	3	4
y	−3	−6	−10	−15

−3 −4 −5

As x increases by 1, y decreases by a different amount each time. The rate of change is not constant, so this function is nonlinear.

Exercises

Determine whether each table represents a *linear* or a *nonlinear* function. Explain.

1.

x	3	5	7	9
y	7	9	11	13

2.

x	1	5	9	13
y	0	6	8	9

3.

x	3	6	9	12
y	2	3	4	5

4.

x	−2	−3	−4	−5
y	−1	−5	9	8

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Skills Practice

Linear and Nonlinear Functions

Determine whether each table represents a *linear* or a *nonlinear* function. Explain.

1.

x	1	2	3	4
y	8	12	16	20

2.

x	0	2	4	6
y	5	3	0	−4

3.

x	−3	−5	−7	−9
y	5	9	13	17

4.

x	3	1	0	−2
y	7	7	7	7

5.

x	3	0	−3	−6
y	1	6	11	16

6.

x	−1	0	1	2
y	−2	0	2	4

7.

x	1	2	3	4
y	5	7	9	11

8.

x	−2	0	2	4
y	0	1	3	9

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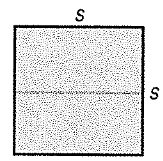
Problem-Solving Practice

Linear and Nonlinear Functions

GEOMETRY For Exercises 1 and 2, use the following information.

Recall that the perimeter of a square is equal to 4 times the length of one of its sides, and the area of a square is equal to the square of one of its sides.



1. Write a function for the perimeter of the square. Is the perimeter of a square a linear or nonlinear function of the length of one of its sides? Explain.

2. Write a function for the area of the square. Is the area of a square a linear or nonlinear function of the length of one of its sides? Explain.

3. **BUSINESS** The Devon Tool Company uses the equation $p = 150t$ to calculate the gross profit p the company makes, in dollars, when it sells t tools. Is the gross profit a linear or nonlinear function of the number of tools sold? Explain.

4. **GRAVITY** A camera is accidentally dropped from a balloon at a height of 300 feet. The height of the camera after falling for t seconds is given by $h = 300 - 16t^2$. Is the height of the camera a linear or nonlinear function of the time it takes to fall? Explain.

5. **LONG DISTANCE** The table shows the charge for a long-distance call as a function of the number of minutes the call lasts. Is the charge a linear or nonlinear function of the number of minutes? Explain.

Minutes	1	2	3	4
Cost (¢)	5	10	15	20

6. **DRIVING** The table shows the cost of a speeding ticket as a function of the speed of the car. Is the cost a linear or nonlinear function of the car's speed? Explain.

Speed (mph)	70	80	90	100
Cost ($)	25	50	150	300

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4.1 Equations of Linear Functions

In this section we'll still be using the slope-intercept form of proportional function equations. Let's take another look at defining the variables so we don't confuse the initial value for one of the variables we are comparing in the proportional relationship.

Defining the Variables and Writing Equations

We want to make sure we can correctly identify the initial value in each situation so that we define our variables correctly. The key to equations is finding the initial value first. This tells us what our dependent variable is. Then we look for the lowest terms proportion ratio or rate of change to use for our slope.

Example 1: *A company gained value at the rate of* $\$200{,}000$ *per day after its IPO (initial public offering) of one million dollars.*

Notice that the initial public offering is where the company started at. That is the initial value. As such, the $\$1{,}000{,}000$ tells us that the value of the company in dollars is going to be our independent variable. We can go ahead and define that now.

$$v = value\ of\ company\ in\ dollars$$

Now we ask, what is that dollar amount being compared to? The hint is in the word "rate" which compares the value in dollars and the number of days after the company's opening to the public stock market. That means we could define the independent variable as follows:

$$d = days\ since\ the\ IPO$$

Finally, we put it all together to make our equation. We know the value depends on the number of days since the initial public offering, so we substitute our slope and initial value into the generic form of a linear functions which is $y = mx + b$ as follows.

$$v = 200{,}000d + 1{,}000{,}000$$

Example 2: *To be a member at a gym you must pay a one-time* $\$25$ *entry fee plus* $\$10$ *per month.*

First, what is the initial value? There is a one-time fee of $\$25$ that is the initial value. This means that dollars, in this case the cost, is our dependent variable. We can define it as follows:

$$c = cost\ of\ gym\ membership$$

Next, think of the slope or rate of change in this problem. How does the rate change? It changes by $\$10$ per month. The entry fee does not change and is not part of the rate of change. What changes is how much you pay when compared to the number of months you sign up for at the gym. That means we are comparing the amount you pay and the number of months making the number of months the independent variable. We can define the independent variable and write the equation based on that as follows:

$m = number\ of\ months\ for\ membership$ $\qquad\qquad c = 10m + 25$

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What do we do if there is no initial value? How can we tell which is the independent and which is the dependent variable? Consider the following example.

Example 3: *To buy an all access pass at the local concert hall you must pay* $\$200$ *for every* 3 *months.*

First, what is the initial value? We don't have one, but we do have a rate of change that tells us we are comparing money (or cost) to the number of months you buy a pass for. Without an initial value, we could officially let either of those be the independent and dependent variable, but it does make more sense that the total cost would depend on the number of months you buy the pass for. Therefore, we'll define the variables and equation as follows:

$$c = cost\ of\ all\ access\ pass$$

$$m = \#\ of\ months\ to\ use\ all\ access\ pass$$

$$c = \frac{200}{3}m$$

Again, without the initial value, we could set up the equation in the opposite way as $m = \frac{3}{200}c$, but that intuitively may feel backwards.

One last warning: there could be extra information in the problems that are not necessary. Just because a number is there, doesn't mean it has to be used. Think about whether or not it fits the problem and has to do with what you are trying to solve.

Solving Linear Equations

Let's solve some proportional function problems that have initial values using equations. Consider the gym membership situation from above. We could ask how much it would cost to be a member for one year. That would mean that $m = 12$ since one year is twelve months. We'll substitute and solve as follows:

$$c = 10m + 25$$

$$c = 10(12) + 25$$

$$c = 120 + 25$$

$$c = 145$$

This means that the cost for one year would be $\$145$ total. We could also solve this problem if we knew the cost and not the number of months. For example, how many months of membership could you buy if you had $\$365$? That would mean that $c = 365$, so we'll substitute and solve as follows:

$$c = 10m + 25$$

$$365 = 10m + 25$$

$$365 - 25 = 10m + 25 - 25$$

$$365 - 25 = 10m \cancel{+ 25 - 25}$$

$$340 = 10m$$

$$\frac{340}{10} = \frac{10m}{10}$$

$$\frac{340}{10} = \frac{\cancel{10}m}{\cancel{10}}$$

$$34 = m$$

So we could buy 34 months of membership with $365.

Finding the Initial Value (y-intercept) or Rate of Change (Slope)

Pretend that a company gained value by $100 per day and after 7 days was worth $850. What was the initial value of the company? Let's start by defining the variables and writing an equation as best as we can. We know we are comparing the value of the company and the number of days. So we might do this:

$$v = value\ of\ company\ in\ dollars$$

$$d = days\ since\ the\ IPO$$

$$v = 100d + b$$

Notice that we don't know the initial value because that's what the question asked us to find. Therefore we'll just leave the variable b in the equation as a place holder for our initial value. However, we do know that after 7 days, or when $d = 7$, the company was worth $850, or $v = 850$. Let's substitute those values in and solve for the initial value, or b.

$$v = 100d + b$$

$$850 = 100(7) + b$$

$$850 = 700 + b$$

$$850 - 700 = 700 + b - 700$$

$$850 - 700 = \cancel{700} + b \cancel{- 700}$$

$$150 = b$$

Therefore the initial value of the company was $150.

Similarly we could solve for the slope, or rate of change, in a problem. Consider an appliance salesman who gets paid $50 every day plus some unknown amount for every appliance they sell that day. Let's say the appliance salesman made $475 after selling 17 appliances. How much does he get paid per appliance? Again, we'll define our variables and write the equation first.

$$p = total\ pay$$

$$a = number\ of\ appliances\ sold \qquad\qquad p = ma + 50$$

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Notice that the \$50 every day is an initial value for the pay each day. That never changes. We also left the m in the equation because we don't know the rate he gets per appliance. So let's substitute what know, which is that when $a = 17$ then $p = 475$, and solve for m.

$$p = ma + 50$$

$$475 = m * 17 + 50$$

$$475 = 17m + 50$$

$$475 - 50 = 17m + 50 - 50$$

$$475 - 50 = 17m \cancel{+ 50 - 50}$$

$$425 = 17m$$

$$\frac{425}{17} = \frac{17m}{17}$$

$$\frac{425}{17} = \frac{\cancel{17}m}{\cancel{17}}$$

$$m = 25$$

This means that the salesman gets paid \$25 per appliance that he sells.

Comparing Linear Equations

Now we have two values to compare in equations, the rate of change and the initial value. Let's say that a restaurant wants to buy paper plates in bulk so they want to join a wholesale store (like Sam's Club). There are three stores they could join:

Store 1: Charges a membership fee of \$100 and charges \$25 per bulk package of paper plates.

Store 2: Charges a membership fee of \$50 and charges \$30 per bulk package of paper plates.

Store 3: Charges a membership fee of \$200 and charges \$20 per bulk package of paper plates.

Let's start by defining variables and writing equations for each store.

$$c = total\ cost$$

$$p = number\ of\ bulk\ packages\ of\ paper\ plates\ purchased$$

Store 1: $c = 25p + 100$ **Store 2:** $c = 30p + 50$ **Store 3:** $c = 20p + 200$

Now it would be difficult to ask which store is cheapest because that depends on how many packages of paper plates you buy. However, we can ask which store has the cheapest rate for packages of paper plates. Which one does? Yes, Store 3 with the price of \$20 per package.

Which store has the highest membership fee? That would be Store 3 as well since it charges \$200.

Which store would give the overall cheapest price if the restaurant needed to buy two packages of paper plates? What if it were ten packages? What if it were twenty packages? Substitute each of those values in for p and solve for c.

Two packages

Store 1: $c = 25(2) + 100$

$c = 150$

Store 2: $c = 30(2) + 50$

$c = 110$

Store 3: $c = 20(2) + 200$

$c = 240$

In this case Store 2 is the best store to buy from.

Ten packages

Store 1: $c = 25(10) + 100$

$c = 350$

Store 2: $c = 30(10) + 50$

$c = 350$

Store 3: $c = 20(10) + 200$

$c = 400$

In this case there is a tie between Store 1 and Store 2.

Twenty packages

Store 1: $c = 25(20) + 100$

$c = 600$

Store 2: $c = 30(20) + 50$

$c = 650$

Store 3: $c = 20(20) + 200$

$c = 600$

In this case there is a tie between Store 1 and Store 3. You should be able to extrapolate (think ahead) and see that if the restaurant needs to buy more than twenty packages of paper plates in bulk, Store 3 will always be cheaper. Can you explain why?

Proportion = Linear without Initial Value

We should note that linear functions can also be proportional functions. You may recall that a proportion typically looks like one fraction equal to another fraction: $\frac{a}{b} = \frac{c}{d}$. You might also note that the two fractions there look nothing like our slope-intercept form of a linear equation which is $y = mx + b$. Let's see if we can reconcile that fact first.

First think of a proportional relationship such as the farmer being able to gather 25 gallons of milk daily for every 3 cows. That means if there are zero cows, there is zero milk. In other words the initial value is zero. Last unit we could have written the equation for this relationship as: $g = \frac{25}{3}c$ where g is the number of gallons of milk the farmer gathers daily and c is the number of dairy cows he has. That equation is definitely not a fraction equals a fraction which is typically how we think of proportions.

However, the point of a proportion is to have two equivalent ratios where each ratio is a comparison of two values. In other words, we typically have a proportion that reads something like, "The number of gallons of milk gathered daily (g) compared to the number of dairy cows (c) is 25 to 3." We might write that proportion as follows: $\frac{g}{c} = \frac{25}{3}$. Now the question is whether or not the two equations are the same. Let's see if we can transform the proportion into the linear equation.

$$\frac{g}{c} = \frac{25}{3}$$

$$c * \frac{g}{c} = \frac{25}{3} * c$$

$$\cancel{c} * \frac{g}{\cancel{c}} = \frac{25}{3} * c$$

$$g = \frac{25}{3} c$$

Thus we see that every proportional function can be written as a linear function. Since this is true, we may as well stick with our familiar linear function form, $y = mx + b$, except that for proportional functions the b value will be zero. This means that the lowest terms proportion ratio is really just the slope of linear function.

Is it Proportional?

If a proportional function is a linear function with an initial value of zero, we can now test if an equation is proportional as well as linear by looking for that initial value. Consider the following three equations and decide if each is proportional as well as linear. In each equation, c is the cost of fixing a leaking ridge cap on a roof based on f, the length in feet that needs to be fixed.

Company 1	**Company 2**	**Company 3**
$c = 12f$	$c = 10f + 50$	$c = 15f$

Which of these have no initial value? It looks like Company 1 and Company 3 have no initial value thereby making them proportional. In other words the ratio of cost to feet fixed is 12 to 1 for Company 1 and 15 to 1 for Company 3.

Company 2 is not proportional because there is an initial value (perhaps a labor fee) of 50. It's still linear because it is in the form $y = mx + b$, where the slope is $\frac{10}{1}$, but the initial value makes it not proportional.

Lesson 4.1

Define variables and create equations for each of the following linear situations.

1. It will cost \$45 to replace the chain on your bicycle plus \$15 per hour of labor.

2. It takes you 11 minutes for every 2 miles you run.

3. You spend 5 minutes on every 2 questions on the test.

4. It takes you 9 minutes for every 2 toilets to clean, and you have already spent 45 minutes cleaning the house.

5. Your parents pay you \$5 for every hour you babysit.

6. The musical cast started with \$1200 in donations and earns \$45 for every 6 tickets sold.

7. At the beginning of the year, you receive 20 free participation points. You can lose 2 participation points every time you forget to bring your supplies to class.

8. Student council ordered one pizza for every 4 students that are attending the after school dance.

9. Which of the problems above are proportional and how do you know?

Use the given equation to solve the linear questions.

10. If a roller coaster starts 12 meters above the ground and climbs 2 meters every second (s), the roller coaster's height (h) would be based on the equation $h = 2s + 12$. How long would it take to reach the top of the hill that is 80 meters above the ground?

11. If it is going to cost you \$525 dollars to start a lawn care business with your friend, but you will earn an average of \$73 for every 4 yards (y), your profit (p) is based on the equation $p = \frac{73}{4}y - 525$. How much profit would you make if you were scheduled to mow 48 yards the first summer?

12. It was raining at a rate of 1 inch every 3 hours. If it rained at that constant rate for 6 hours (h), how many inches of rain (r) would there be if you followed the equation $r = \frac{1}{3}h$?

13. It costs \$550 for buses to transport students to the C. A. N. D. L. E. S. Holocaust Museum in Terre Haute, Indiana. If the museum charges \$5 for every 2 students (s), the total cost (c) of the trip is based on the equation $c = \frac{5}{2}s + 550$. How much would it cost to bring 196 students?

14. School policy states that there must be one teacher for every 24 students. If there are 120 students attending the field trip, how many teachers would be necessary to chaperone if you followed the equation $t = \frac{1}{24}s$?

15. If you spent \$10.35 total (t) purchasing songs online for \$1.15 each, how many songs did you buy (s) if you followed the equation $t = 1.15s$?

16. If a rose bush is planted when it is 14 inches tall and it grows three inches every five days (d), it's height (h) is based on the equation $h = \frac{3}{5}d + 14$. How many days would it take for the rose bush to be 50 inches tall?

17. The average computer consumes 130 watts of power per hour. If your energy bill shows you used a total of 845 watts (w) for a single day, for how many hours (h) was your computer running if you followed the equation $w = 130h$.

18. Which of the above problems represent proportional situations and how do you know?

Create an equation to solve the following questions.

19. A mama bird must gather 5 worms for every 2 baby birds in order to provide them with adequate nutrition. If she has 6 baby birds, how many worms must she find?

20. At the Charleston Bowling Lanes, it costs \$2 to rent shoes plus \$1.50 per game of bowling. How many games would you be able to bowl for \$11?

21. In the Tour De France Lance Armstrong pedaled at an average pace of 49 kilometers per hour. If the race is 3479 kilometers long, how long did Lance spend cycling?

22. You've been working on your math homework for 25 minutes already. If it takes about 10 minutes for every 3 problems, how long will you have spent on homework if you only have 6 problems left?

23. A famous fashion designer spent $9.5 million on fabric for her new spring line. If she earns approximately $1.2 million for each dress she sells, how many dresses will she have to sell to make a profit of $14.5 million?

24. Your parents put a down payment on your car, but they are requiring you to pay the monthly payment of $85. If you will have to pay a total of $2125 for the car, how long will it take you to pay it off?

25. The Pick Your Burgers restaurant spent $98,145 on food, utilities, and labor this month. If the average table of 4 customers spent $73, how much profit did the restaurant make if they served 7092 customers?

26. On average, it takes 5 bales of hay to feed 2 horses. If you have 9 horses, how many bales of hay will you have to purchase?

Answer the following questions comparing linear equations and descriptions.

You are deciding which gas company to choose as you travel across the country on a long vacation with your family. Here is the information about the cost (c) for gallons of gas (g) for each company.

Gas Up Charges $4.01 for each gallon of gas	**Automart** Charges $81 for 20 gallons of gas
The Fuel Shop Cost is modeled by the equation $c = 4.03g$	**Full Tank** Cost is modeled by the equation $c = 4.10g$

27. Which company charges the most per gallon of gas? How do you know?

28. Which company charges the least per gallon of gas? How do you know?

29. How much would each company charge you for 12 gallons of gas? Which is the cheapest?

30. If you had $100 to spend on gas, how many gallons could you buy from each gas station?

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Dr. Kal is studying how age and gender affect calorie expenditure. Here is the information about the number of calories burned (c) *based on the number of miles* (m) *walked in a day.*

Paul (25) Burns 1390 calories plus 1040 calories from walking 10 miles	**Ishmael (58)** Burns 1305 calories plus 220 calories from walking 2 miles
Jerika (31) Calorie expenditure is based on the equation $c = 98m + 1225$	**Pamela (62)** Calorie expenditure is based on the equation $c = \frac{205}{2}m + 1189$

31. Who burns the most calories per mile, and how do you know?

32. Who burns the least calories per mile, and how do you know?

33. Who burns the most calories without walking, and how do you know?

34. How far would each person have to walk (to the nearest hundredth) to burn 2000 calories?

35. If each person walks 10 miles, who burns the most calories for that day?

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1.2 Graphing Functions

We can graph functions to get a visual representation of the relationship between two quantities. We graph these on the coordinate plane, but we may not always use the variables x and y.

Input/Output Charts

To graph a function, we first need an input/output chart. This chart will give us the points we need to graph on the coordinate plane. Let's start by graphing the following function:

$$c(t) = 2t + 3$$

For this function, notice that t is the input, or independent variable, and c is the output, or dependent variable. We'll now make a simple chart with five spaces to fill out as follows:

Input t					
Output $c(t)$					

Sometimes input values will be given to us to plug in, other times we will need to make up our own. In this case, we are not given values for the input. Therefore, it is suggested to use the values from -2 to 2 to make sure we get a good picture of the function. It is not always necessary to find five points, but the more points we have, the better graph we will get.

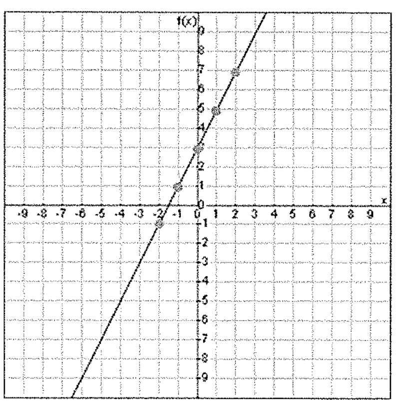
Input t	-2	-1	0	1	2
Output c(t)					

Now we evaluate the function for each input. Let's look at the work for $c(-2)$.

$$c(-2) = 2(-2) + 3 = -4 + 3 = -1$$

Input t	-2	-1	0	1	2
Output $c(t)$	-1	1	3	5	7

Following this same process for each input value, we get the table at the right.



Now we plot each associated input and output as a point like this: $(input, output)$ or $(t, c(t))$. Since t is the dependent variable, that takes the place of x and $c(t)$ will take the place of y. Graph each point and connect the points as we can see at the left.

In most cases the input/output chart only uses the variables as labels instead of "input" and "output". That would look like this:

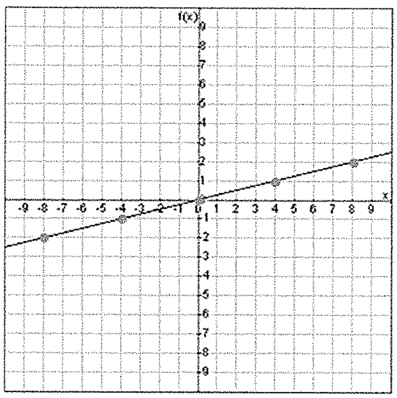
t	-2	-1	0	1	2
$c(t)$	-1	1	3	5	7

Notice we plotted five points: $(-2,-1), (-1,1), (0,3), (1,5)$, and $(2,7)$.

Deciding on Appropriate Inputs

Since we are graphing by hand, it is easiest if we work with integer inputs and outputs. Some functions have fractions, decimals, or even square roots that make our choice of inputs critical. For example, consider the function $a(b) = \frac{1}{4}b$.

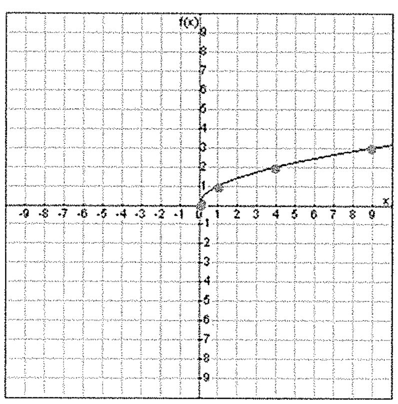
b	-8	-4	0	4	8
$a(b)$	-2	-1	0	1	2



If we choose $b = 1$ as an input, we'll have to graph the point $\left(1, \frac{1}{4}\right)$ which is not convenient by hand. Therefore, we should choose values for b that we can multiply by $\frac{1}{4}$ and get integer outputs for $a(b)$. Perhaps the input/output chart given to the left would work best yielding the graph below the chart.

Notice that choosing multiples of 4 for our inputs allowed integer outputs.

x	0	1	4	9
$f(x)$	0	1	2	3



Let's look at the square root function $f(x) = \sqrt{x}$. Since we can't take the square root of negative numbers, we won't use any negative inputs. Also, since the number 2 does not have an integer square root, we'll skip ahead to the inputs that do have integer square roots. Therefore we might use an input/output chart like the one to the left yielding the graph below the chart.

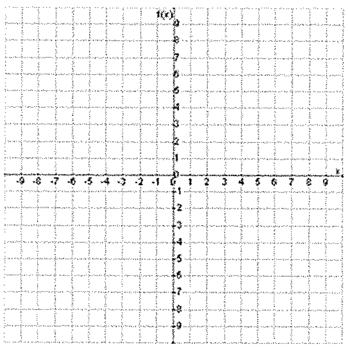
Notice that we only used four inputs instead of five since the next input yielding an integer output would be $x = 16$ and that x value would be off the coordinate plane we have which only goes up to $x = 10$.

Lesson 1.2

Graph the following functions by filling out the table using the given inputs (x values).

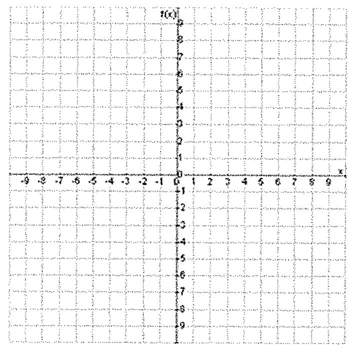
1. $f(x) = x^2 - 7$

x	-2	-1	0	1	2
$f(x)$					



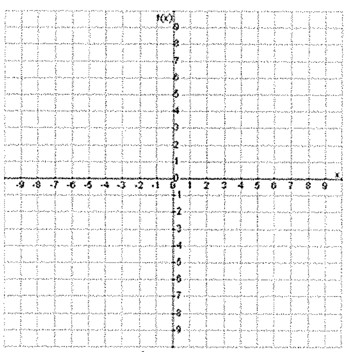
2. $f(x) = \frac{1}{3}x + 2$

x	-6	-3	0	3	6
$f(x)$					



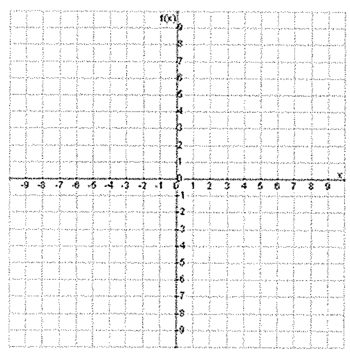
3. $f(x) = \sqrt{x+9}$

x	-9	-8	-5	0	7
$f(x)$					



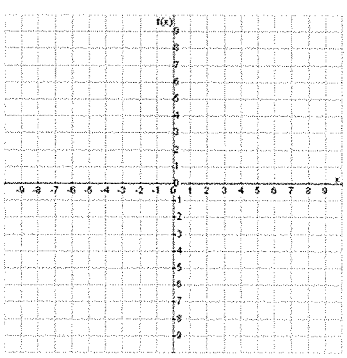
4. $f(x) = 2x^2 - 1$

x	-2	-1	0	1	2
$f(x)$					



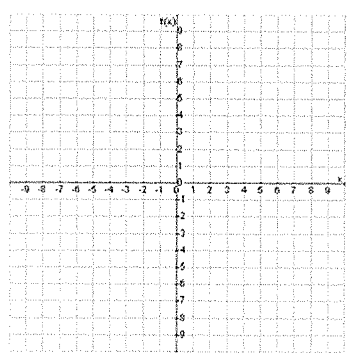
5. $f(x) = \frac{1}{5}x + 2$

x	-10	-5	0	5	10
$f(x)$					



6. $f(x) = \sqrt{x+7}$

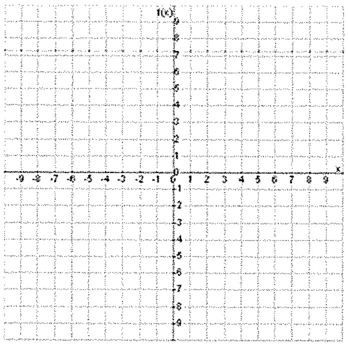
x	-7	-6	-3	2	9
$f(x)$					



Graph the following functions by filling out the table using the inputs (x values) that you think are appropriate.

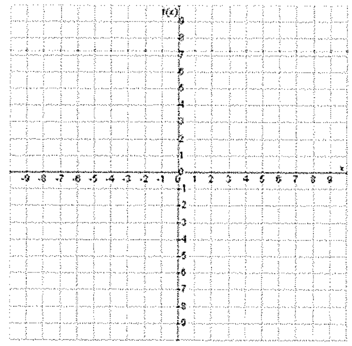
7. $f(x) = 2x^2 - 8$

x					
$f(x)$					



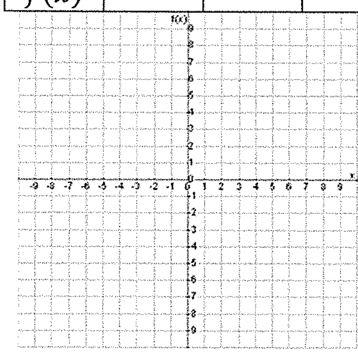
8. $f(x) = \frac{2}{3}x - 4$

x					
$f(x)$					



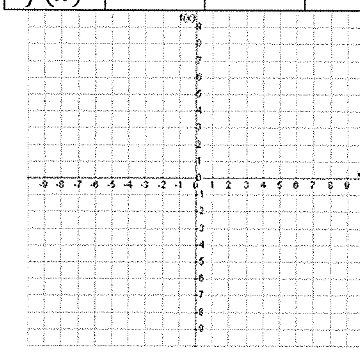
9. $f(x) = \frac{1}{2}x - 4$

x					
$f(x)$					



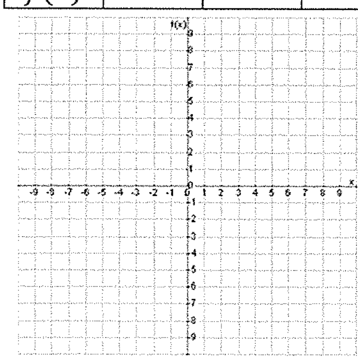
10. $f(x) = \sqrt{x + 8}$

x					
$f(x)$					



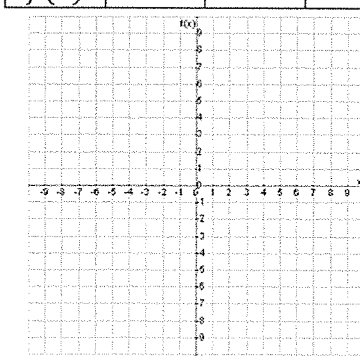
11. $f(x) = -\sqrt{x + 7}$

x					
$f(x)$					



12. $f(x) = -x^2 + 4$

x					
$f(x)$					



13. Explain why it would be beneficial to use the inputs $-2, -1, 0, 1,$ and 2 for the function $f(x) = x^2 + 1$.

14. Explain why it would be beneficial to use the inputs $-8, -4, 0, 4,$ and 8 for the function $f(x) = \frac{3}{4}x - 2$.

15. Explain why it would be beneficial to use the inputs $-9, -8, -5, 0,$ and 7 for the function $f(x) = \sqrt{x + 9}$.

16. Explain how you would choose 5 different inputs for the function $f(x) = \sqrt{x + 6}$. Explain why you feel these are the best input values for this function.

17. For problems 2, 5, 8, 9, describe a pattern in the change in the $f(x)$ values for each function.

18. For problems 2, 5, 8, 9, explain similarities and differences in the structure of the equations.

19. For problems 2, 5, 8, 9, explain similarities and differences in the graph of each function.

4.2 Graphs of Linear Functions

In Unit 3 we graphed functions, so we already know how to graph linear (and proportional) functions. After a review of that, we'll discuss solving with a graph and comparing graphs.

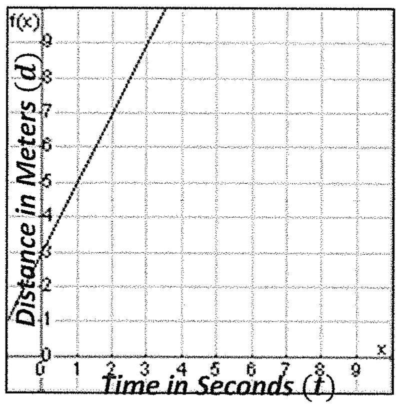
Graphing Linear Functions with Initial Values

If a linear function is given to us as an equation, we simply graph it using an x/y chart like we did previously in Unit 3.

Example 1: *Adam was given a 3 meter head start and runs at 2 m/s which is the equation* $d = 2t + 3$.

Notice in this case we don't have an x and y as the variable, but we know that the variable d is equivalent to the variable y since it is the dependent variable or output and the variable t is equivalent to the variable x since it is the independent variable or input. Thus we can graph as follows:

t	0	1	2	3
d	3	5	7	9



Example 2: *A company uses* 2 *bottles of ink to print* 3 *t-shirts and the machine uses one bottle to warm up.*

Let's start by defining the variables and writing an equation:

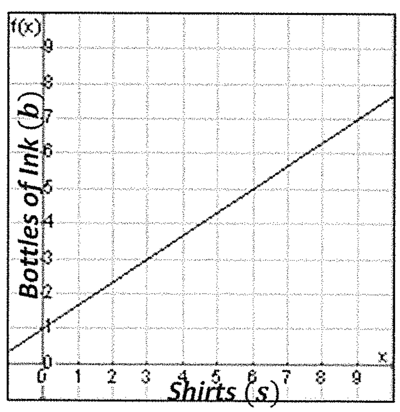
$b = bottles\ of\ ink\ used$

$s = shirts\ printed$

$b = \frac{2}{3}s + 1$

Now graph with an x/y chart as follows:

s	0	3	6
b	1	3	5



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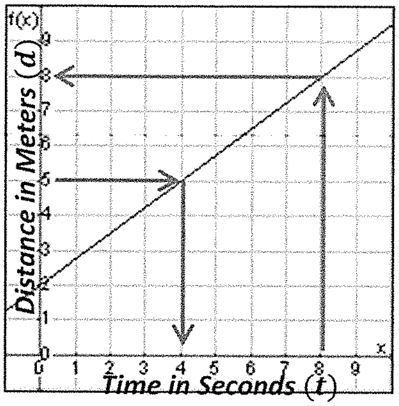
Solving Linear Situations with Graphs

If we are given a graph with an initial value, we can still solve linear problems. Consider the following graph and answer the question, "How far did the toy car travel after 8 seconds?"

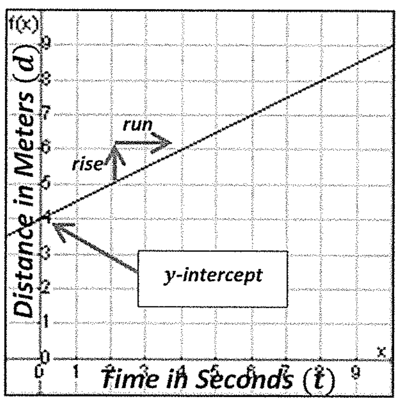


To solve using the graph, simply go to 8 seconds and move up to the line. How far did the car travel? It looks like the toy car has traveled 8 meters at that point.

Similarly we could ask a question such as, "How long did it take the toy car to travel 5 meters?" Go up to five meters and move over to the line. Looking straight down from that point we see that it took the toy car 4 seconds.

Getting Equations from Graphs with Initial Values

We can still get equations from graphs with initial values, but it will take the extra step of looking for the y-intercept on the graph.



First, find the slope using two points from the graph. Notice that on this graph we have a point on the line at (2,5) and (4,6). Those are not the only two nice integer value points on this line, but we only need two. Since the slope can be thought of as $\frac{rise}{run}$, we simply look for how far the line rises and runs between those two points.

It goes from 5 meters to 6 meters, so that is a rise of 1. It goes from 2 seconds to 4 seconds which means a run of 2. Therefore the rise over the run is $\frac{1}{2}$. This is our slope, rate of change, or lowest terms proportion ratio (whatever we want to call it).

We are still missing the y-intercept, or initial value, but we can clearly see it on the graph. The line crosses the y-axis at 4, so our initial value is 4. Now we can write the equation: $d = \frac{1}{2}t + 4$. What if we didn't have the whole graph, but only those two points to work with? Let's zoom in and find out.

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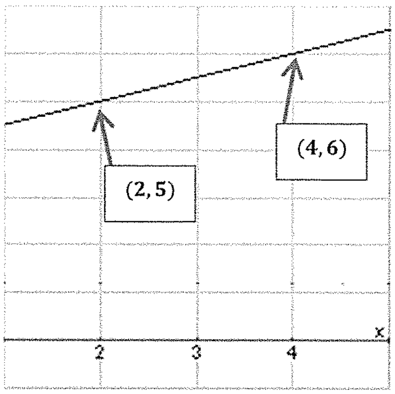


We still have our two points and therefore can find the slope, but we can't see the initial value. However, since we know the slope and a point on the line, we can substitute those values in to find the initial value.

$$d = mt + b$$

Substitute the slope

$$d = \frac{1}{2}t + b$$

Substitute one point

$$6 = \frac{1}{2}(4) + b$$

$$6 = 2 + b$$

Solve for initial value

$$6 - 2 = 2 + b - 2$$

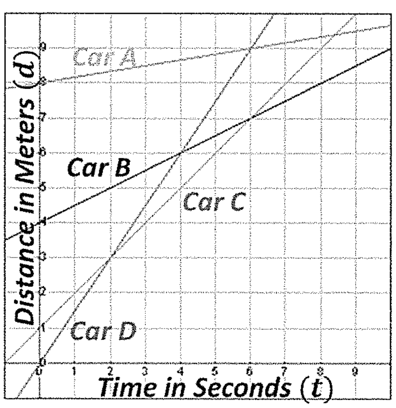
$$4 = b$$

Write the equation

$$d = \frac{1}{2}t + 4$$

Comparing Linear Function Graphs

We may need to compare graphs to other graphs or compare graphs to equations. Just as with equations, one of the main comparison points is the slope of the graph. For example, looking at the following graph, we could ask which toy race car is faster.



To compare these race cars, we need to know their speeds. The speed is a measure of distance over time, which is the slope or steepness of each line. So let's find the slope for each race car. After finding the rise and run for each car, we should come to the following values:

Car A: $\frac{1}{6}\ m/s$ **Car B:** $\frac{1}{2}\ m/s$ **Car C:** $1\ m/s$ **Car D:** $\frac{3}{2}\ m/s$

Based on this information, we see that Car D is the fastest car. We also know that Car A is the slowest. The curious thing is that Car A, even though it is the slowest, stays above most of the lines for the majority of the graph. This is because of the initial value for each car.

The initial value effectively is a head start for each of the cars. Notice the following initial values:

Car A: $8\ m$ **Car B:** $4\ m$ **Car C:** $1\ m$ **Car D:** $0\ m$

The reason the slowest car is ahead in the race for most of the graph is because of its tremendous head start. It received a whopping 8 meter head start! Also note that Car D did not have a head start and is therefore proportional.

We could also ask lots of other interesting questions now. For example, if the race were only 5 seconds long, which car would travel the farthest? Go to 5 seconds on the graph and move up. Which car is highest? Car A (again, due to the head start).

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If the race were 10 meters long, which car would get to the finish line first? Go up to 10 meters and look over. Which car do you hit first? Car D finishes somewhere between 6 and 7 seconds.

If the race were 10 meters long, what order would the cars finish in? Go up to 10 meters and look over. Car D finishes first and then Car C, but we're not sure when Car A and B finish. It turns out that Car A and B would tie for last place at 12 seconds. Can you prove it?

Lesson 4.2

Create a graph for each of the following linear situations or equations.

1. At Pizza Hut It costs \$8 for each large pizza plus \$5 for a delivery tip.

2. It costs \$40 per ticket to Six Flags plus \$100 for gas there and back.

3. The number of lives (l) based on the number of levels completed (c) is determined by the following equation: $l = \frac{1}{3}c + 4$

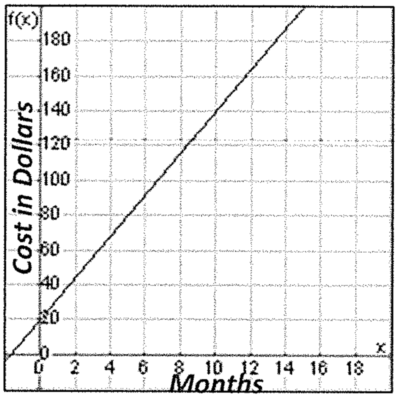
4. When making fudge, four ounces of sugar are needed for every ounce of chocolate.

5. For every 2 green peppers used in a salsa there are 3 red peppers used.

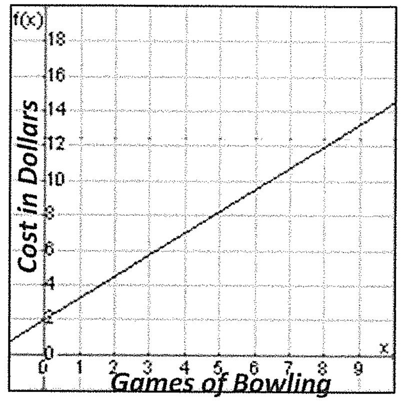
6. Mutant alien frogs from Zappax have a number of feet (f) based on the number of toes they are born with (t) according to the following equation: $f = \frac{1}{7}t$.

Use the given graph to solve the linear questions.

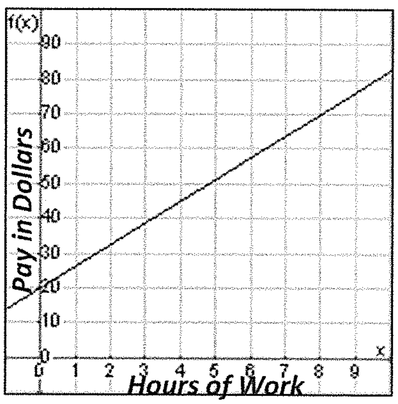
7. How much will it cost for ten months of internet service?



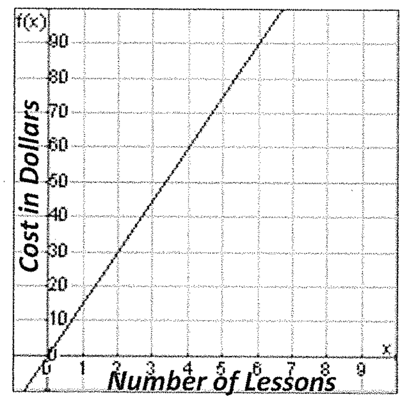
8. How many games of bowling can you play if you can spend $12?



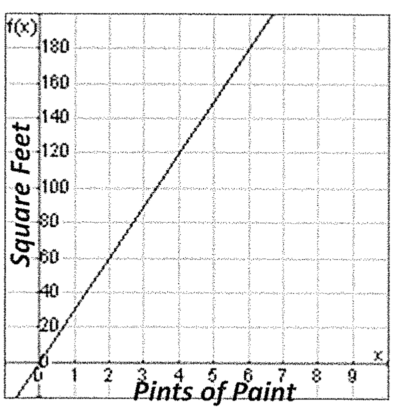
9. How many hours would you have to work to earn $70?



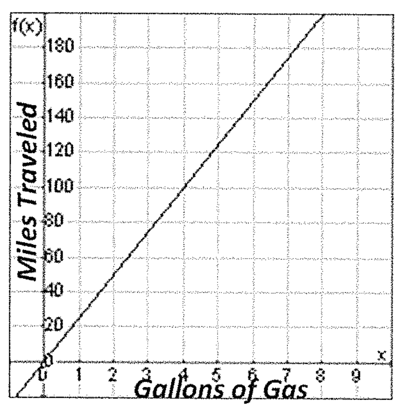
10. How much would it cost for four lessons?



11. How many pints of paint should you buy if you have to paint 120 square feet?



12. How many miles can you travel if you have four gallons of gas left in your tank?



13. Which of the above graphs are proportional situations and how do you know?

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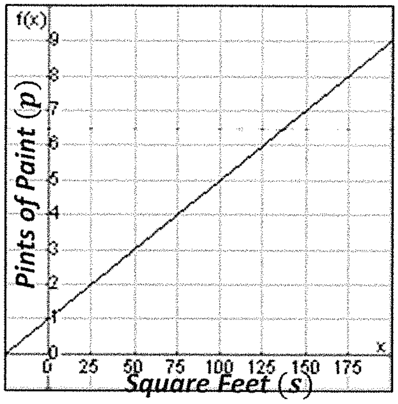
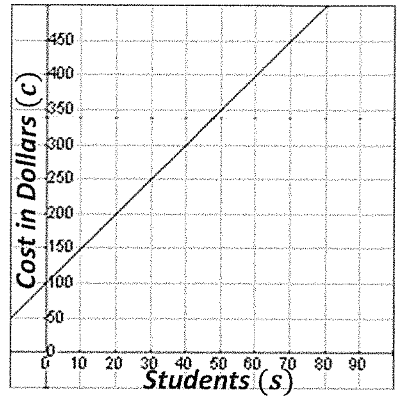
Create an equation for the following linear graphs.

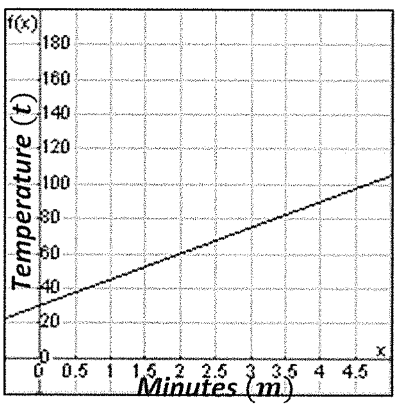
14. Number of pints of paint (p) needed for a certain number of square feet (s)



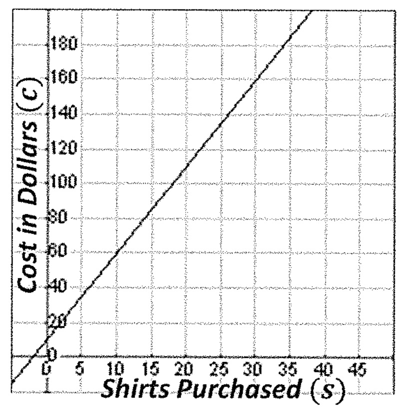
15. The cost (c) of a field trip based on the number of students (s) attending



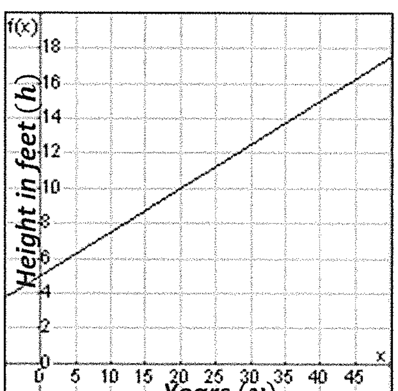
16. Temperature (t) of water per minute (m) of time on the stove



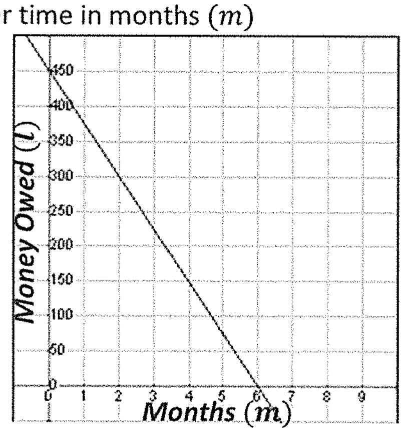
17. Cost (c) of an order depending on the number of shirts (s) purchased



18. A tree's height (h) based on the number of years (y) since being transplanted



19. Money you owe on your loan (l) for your first car over time in months (m)



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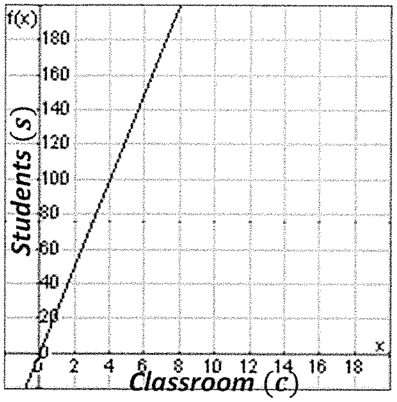
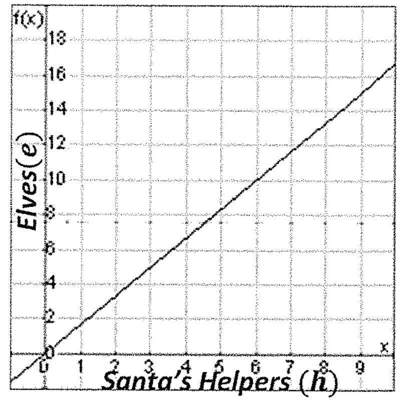
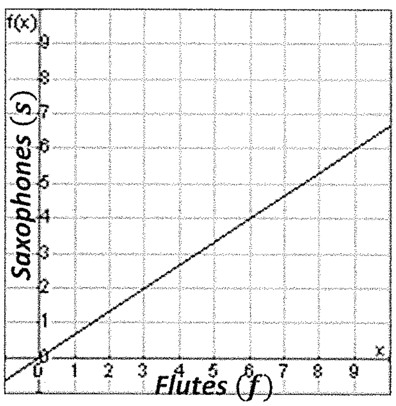
20. Number of students (s) in every classroom (c)



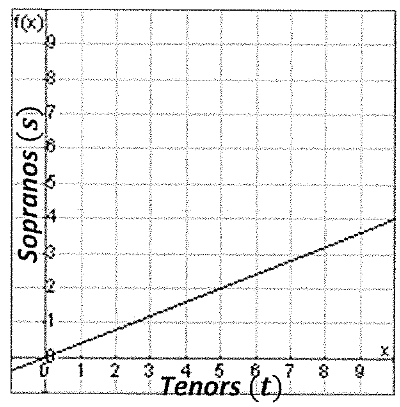
21. Number of elves (e) for Santa's helpers (h)



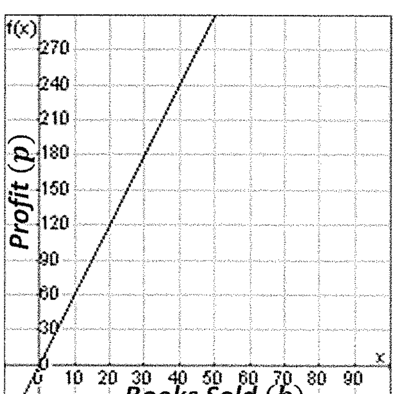
22. Number of saxophones (s) compared to the number of flutes (f) in an orchestra



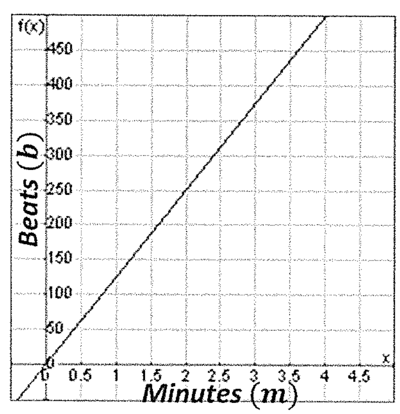
23. Number of sopranos (s) compared to the number of tenors (t) in a choir.



24. Amount of profit (p) based on the number of books sold (b)



25. Number of beats (b) per minute (m) in a hip-hop song



26. Which of the graphs from problems 14 to 25 are proportional and how do you know?

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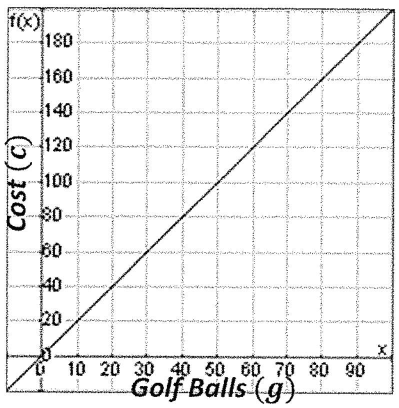
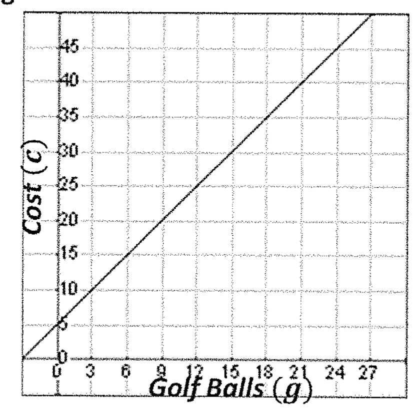
Answer the following questions comparing linear function equations, graphs and descriptions.

Various golf ball manufacturers offer deals for packs of golf balls. Here is the information about the total cost (c) for golf balls (g) including shipping costs.

Callaway Charges a fee of $10 for shipping and $5 for 3 golf balls	**Nike** Cost is modeled by the equation $c = \frac{5}{2}g + 5$
Titleist 	**Top-Flight** 

27. Which manufacturer has the cheapest cost per golf ball, and how do you know?

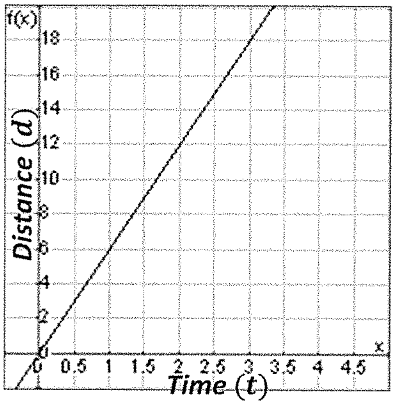
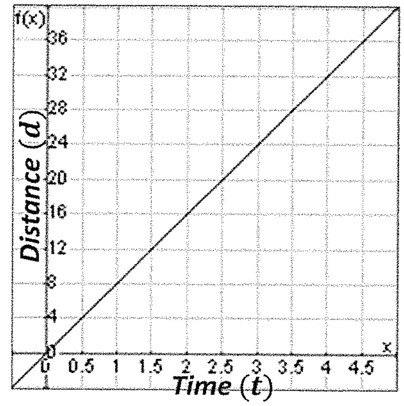
28. Which manufacturer has the cheapest shipping fee, and how do you know?

29. How many golf balls could you buy at each company for $200? Which manufacture would give you the most golf balls for that amount of money?

30. Which manufacturer would be the cheapest if you wanted to buy 30 golf balls?

Answer the following questions comparing proportional function equations, graphs and descriptions.

Scientists are studying how location affects the speed of a bottlenose dolphin. Here is the information about the distance (d) in kilometers a dolphin traveled in terms of time (t) in hours.

Dolphin in Gulf of Mexico Swims 11 kilometers in 2 hours	**Dolphin in Mediterranean Sea** Distance is modeled by the equation $d = \frac{35}{4}t$
Dolphin in Indian Ocean 	**Dolphin in North Atlantic Ocean** 

31. Which location has the fastest dolphin?

32. Which location has the slowest dolphin?

33. How far could each dolphin travel in 4 hours? Which location has the dolphin that went the farthest?

34. How long would it take each dolphin to swim 100 kilometers? Which location has the dolphin that finished in the shortest amount of time?

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4.3 Tables of Linear Functions

The final concept we'll cover this unit is the table form of linear functions. Just like in the previous sections, some of these linear functions may also be proportional which means there will be no initial value. We'll start by assuming there is an initial value..

Filling Out a Table from Equations and Graphs

Perhaps the simplest thing that we can do is fill out a table based on an equation or a graph. Since the table is designed to look at specific input/output pairings, we may need to pick appropriate inputs just like we did when graphing functions. Let's fill out the tables for the following examples.

Example 1: $d = 5t - 3$

t	0		2		4
d		2		12	

For this table, note that some of the values have been given for us. If we are given an input (t in this case), then simply input that into the equation to find the output. For example, we have $t = 0$ as an input, so substitute in as follows: $d = 5(0) - 3$. Multiplying five by zero and then subtracting three gives us $d = -3$.

t	0		2		4
d	-3	2		12	

If we are given an output (d in this case), substitute that into the equation and solve for the input. For example, note that we are given $d = 2$ as an output. Substitute into the equation and solve as follows:

$$d = 5t - 3$$

$$2 = 5t - 3$$

Adding three to both sides and then dividing by five gives us that $t = 1$.

t	0	1	2		4
d	-3	2		12	

It also works to look for patterns. For example, we see that the d values are going up by fives, so the next gap for d should be 7, then the 12 is given to us, and the last should be 17. We similarly know that the missing t value is 3. So our final table filled out (with our solutions in red) should look like this:

t	0	1	2	3	4
d	-3	2	7	12	17

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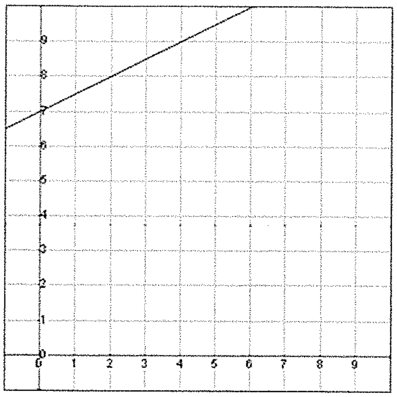
Example 2:



In this example, we aren't given axis labels, so we'll use the standard x and y variables. Here's the table we want to fill out:

x	2		6		10
y		9		11	

For the x values (the input), we can guess from the pattern that we're counting by twos. Can you think of a reason for this based on the graph?

x	2	4	6	8	10
y		9		11	

Now that we have all the inputs, let's get the outputs (y values). At an input of 2, the output on the graph is 8. At an input of 6, follow the graph up to see that the output is 10. However, the input of 10 is off the graph. What will we do? Look for a pattern! Notice that the y values are going up by 1 for every 2 in the x direction. Following this pattern, we know that the last output should be 12.

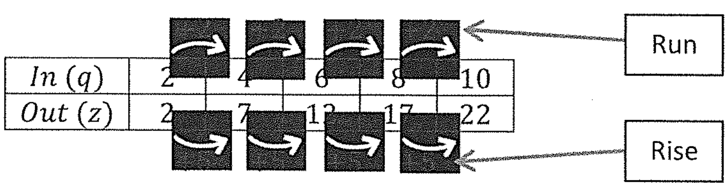
x	2	4	6	8	10
y	8	9	10	11	12

Writing the Equation of a Table

To get a fuller picture of a function, we may want to look at the equation of the function. Since we are currently talking about linear functions with an initial value, we are dealing with equations that will be in the form $y = mx + b$ where m is the slope, rate of change, or lowest terms proportion ratio and b is the initial value or y-intercept. To get the slope, we need the rise (the change in the output) and the run (the change in the input). Examine the following table to see if you can identify the slope.

$In\ (q)$	2	4	6	8	10
$Out\ (z)$	2	7	12	17	22

Check the difference between each of the adjacent outputs (outputs that are next to each other) for the rise. Check the difference between each of the adjacent inputs for the run.



Now we see that the rise is five and the run is two. That means that our slope is $\frac{5}{2}$ and our equation is $z = \frac{5}{2}q + b$ so far, but we still need the initial value.

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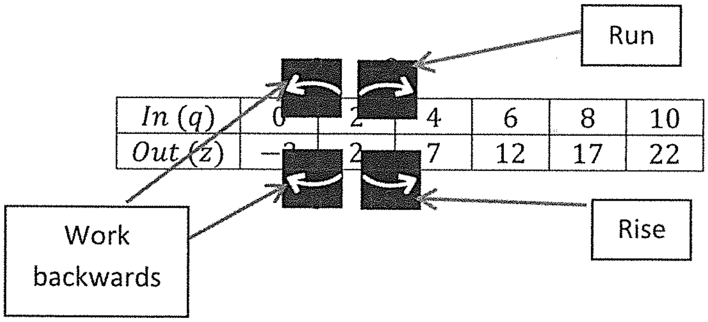
To find the initial value in this table, we need to find what the output is when the input is zero. We can either extend the table backwards following the pattern or solve the equation. Let's first extend the table backwards. Since the run is $+2$, to move backwards we'll go -2 for the input. Since the rise is $+5$, to move backwards on the table, we'll go -5 for the output. That will look like this:



This shows us that the output at input zero (or initial value) is -3. Now we can finish our equation meaning that $z = 5q - 3$.

There may be times when it would take too long to count backwards on the table. For those times, just use the slope and a single input/output pair. Substitute all those values into the generic linear form $y = mx + b$ and solve for b. For example, we know the slope is $\frac{5}{2}$ and the output is $z = 2$ for an input of $q = 2$. Plug all those values (using the proper variables as input and output) in as follows:

$$z = mq + b$$

$$2 = \frac{5}{2}(2) + b$$

$$2 = 5 + b$$

Subtracting five from both sides of the equation shows us that $b = -3$ just like we found earlier.

Remember that in some cases the initial value will be zero making it a proportion. If you work backwards in a table and find an initial value of zero, then you'll know it is a proportion.

Solving Table Problems

Sometimes the answer is right there in the table, and other times we'll have to do some digging to find the answer. Consider the following table that shows the total cost (c) when buying hairless wildebeests (w) after paying the registration fee with the federal government to own exotic pets. How much was the registration fee? To answer this question we'll want to first figure out how much each hairless wildebeest costs by finding the rate of change (or slope). Find the rise and run like normal.

w	12	14	16	18	20
c	\$1900	\$2200	\$2500	\$2800	\$3100

Did you find that the rise was \$300 for a run of 2 wildebeests? This reduces to \$150 per wildebeest. Now we could extend the table all the way back to zero wildebeests to find the registration fee (which is the initial value), but it's probably easier to plug everything we know into an equation as follows:

$$c = mw + b$$

We know it's \$150 per wildebeest, that that is our m value. We also have an input of $w = 12$ giving us an output of $c = \$1900$. Now we'll substitute and solve:

$$1900 = 150(12) + b$$

$$1900 = 1800 + b$$

From here we can see that $b = \$100$ by subtracting 1800 from both sides of the equation. That means that the initial value, or registration fee in this case, was \$100.

Comparing Tables with Initial Values

To compare linear functions in table form, we need their slope and initial values. Consider the following three tables each representing stores that sell fire-breathing beetle wings and decide which store sells beetle wings (w) for the cheapest price (p). Other store charges an entry fee as well.

Hogwarts

w	2	4	6	8	10
p	\$18	\$21	\$24	\$27	\$30

Diagon Ally

w	1	2	3	4	5
p	\$7	\$9	\$11	\$13	\$15

Your Mom's Shop

w	3	6	9	12	15
p	\$13	\$16	\$19	\$22	\$25

Finding the rise and run for each store gives us that Hogwarts charges \$1.50 per beetle wing, Diagon Ally charges \$2 per beetle wing, and Your Mom's Shop charges \$1 per beetle wing.

By either extending the table backward or using the equation method you should find that Hogwarts charges an entry fee of \$15, Diagon Ally charges \$5, and Your Mom's Shop charges \$10 for entry.

If you had to buy 20 beetle wings, which store would be cheapest? Find the equations or extend the tables to see that the Hogwarts total price would be \$45, it would be \$45 at Diagon Ally, and would only be \$30 at Your Mom's Shop.

Lesson 4.3

Create a table for each of the following linear situations, equations or graphs.

1. Game Start charges \$45 (c) for 2 video games (g) purchased.

g	4		12		20
c		\$180		\$360	

2. Susie's hair is 16 inches long and it grows 2 inches in length (l) every 3 months (m)

m					
l					

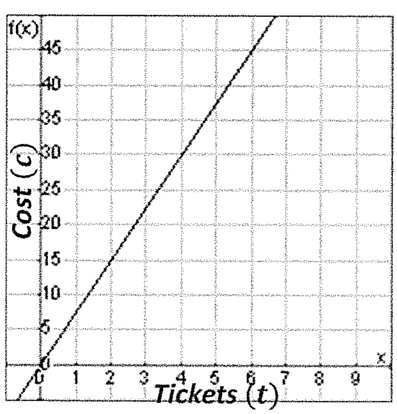
3. The height (h) of a tree in feet after a number of years (y) is determined by the following equation: $h = \frac{1}{3}y + 7$.

y	3		9		15
h		9		11	

4. At Peter's Pizza Palace the total cost (c) for pizzas (p) can be determined by using the following equation: $c = \frac{9}{2}p$.

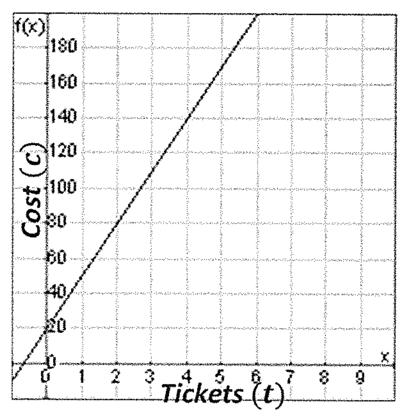
p					
c					

5. The graph shows the total cost (c) per ticket (t) for a student to go to the movies.



t	2		6		10
c		\$30		\$60	

6. The graph shows the cost (c) for tickets (t) to see Taylor Swift in concert if you order tickets online.



t					
c					

Create an equation for the following linear tables.

7. The total cost (c) for miles (m) traveled in a taxi.

m	2	4	6	8	10
c	\$4.50	\$6	\$7.50	\$9	\$10.50

8. The total cost (c) per tournament (t)

t	2	4	6	8	10
c	\$225	\$400	\$575	\$750	\$925

9. The total cost (c) to buy guitar picks (p).

p	5	10	15	20	25
c	\$2	\$4	\$6	\$8	\$10

10. The money earned (m) in a number of weeks (w).

w	2	4	6	8	10
m	\$10	\$20	\$30	\$40	\$50

11. The number of frogs (f) ordered for students (s) in science class.

s	9	15	21	27	33
f	7	9	11	13	15

12. The total cost (c) per hole of golf (g).

g	9	18	27	36	45
c	\$15	\$30	\$45	\$60	\$75

13. The distance traveled (d) in time in hours (h).

h	2	3	4	5	6
d	14	21	28	35	42

14. The amount of profit (p) of a stand selling lemon shake-ups (l).

l	250	300	350	400	450
p	\$50	\$200	\$350	\$500	\$650

15. The total weight of an aquarium (a) holding gallons (g) of water.

g	100	110	120	130	140
a	930	1015	1100	1185	1270

16. The length (l) of a bungee cord that is stretched depending on the weight (w) of the jumper.

w	100	110	120	130	140
l	80	83	86	89	92

17. Which of problems 8 to 16 represent proportions and how do you know?

Use the given tables to solve the linear questions.

18. How many calories (c) would you burn in a day if you walked 2 miles (m)?

m	4	5	6	7	8
c	1800	1900	2000	2100	2200

19. How many minutes (m) would it take for a pot of water to reach a temperature (t) of 210°F?

m	1	2	3	4	5
t	85	110	135	160	185

20. How many cups of cheese (c) would you need for an 18-inch pizza (p)?

p	8	12	16	20	24
c	2	3	4	5	6

21. How many months (m) could you afford the cost (c) of your own cell phone if you have $190?

m	2	4	6	8	10
c	$34	$58	$82	$106	$130

22. How much would it cost (c) to buy 13 shirts (s) at Kohl's?

s	2	4	6	8	10
c	$10	$30	$50	$70	$90

23. How many songs (s) could your purchase for $45 ($c$)?

s	4	6	8	10	12
c	$6	$9	$12	$15	$18

24. How many CDs (c) would an artist need to sell in order to make a profit (p) of $3,000?

c	50	60	70	80	90
p	$2250	$2300	$2350	$2400	$2450

25. How much profit (p) would Harry's Hot Dogs make if they sold 400 hot dogs (h) in a month?

h	200	225	250	275	300
p	100	150	200	250	300

26. Which of problems 18 to 25 are proportional and how do you know?

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Answer the following questions comparing linear function equations, graphs, tables and descriptions.

Your neighborhood friends have decided to have a running race down the street. Here is the information about the distance (d) (including a head start in some cases) in terms of time (t) in seconds.

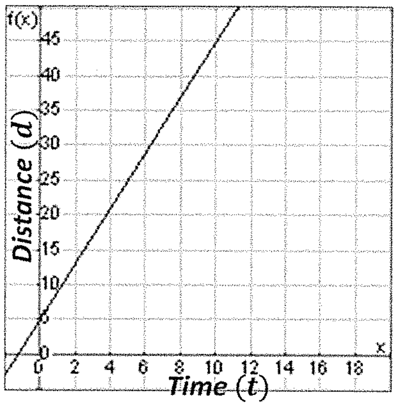
Mitchell

Runs 5 meters in 2 seconds and has a 10 meter head start

Kyra

Distance is modeled by the equation $d = \frac{9}{2}t + 3$

Gloria



Hashim

t	20	22	24	26	28
d	77	84	91	98	105

27. Which runner has the fastest pace, and how do you know?

28. Which runner has the biggest head start, and how do you know?

29. How far could each runner go in 10 seconds? Who would go the farthest?

30. Who would win the race if the race was 15 meters long?

Answer the following questions comparing proportional function equations, graphs, tables and descriptions.

Your family is deciding which activity to participate in while on your vacation in San Diego. Here is the information about the cost (c) for admission for all of your family members (f).

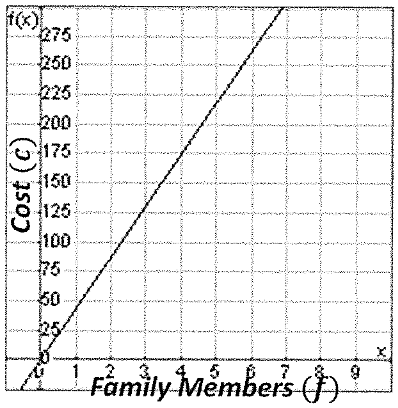
City Tour

Charges $30 per family member

San Diego Zoo

Cost is modeled by the equation $c = \frac{75}{2}f$

SeaWorld



Kayaking

f	2	4	6	8	10
c	65	130	195	260	325

31. Which activity is the cheapest per family member, and how do you know?

32. Which activity is the most expensive per family member, and how do you know?

33. How many people could you bring to each activity if you budgeted $400? Which activity allows you to bring the most people for that amount of money?

34. How much would it cost at each activity to bring a family of 4? Which activity is the cheapest for that many people?

NAME ______________________ DATE ____________ PERIOD _______

Reteach

Linear Functions

A function in which the graph of the solutions forms a line is called a **linear function**. A linear function can be represented by an equation, a table, a set of ordered pairs, or a graph.

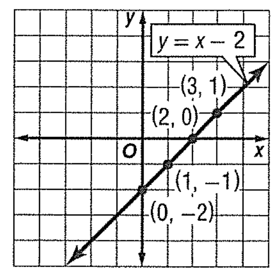
Example 1

Graph $y = x - 2$.

Step 1 Choose some values for x. Use these values to make a function table.

x	$x - 2$	y	(x, y)
0	0 − 2	−2	(0, −2)
1	1 − 2	−1	(1, −1)
2	2 − 2	0	(2, 0)
3	3 − 2	1	(3, 1)

Step 2 Graph each ordered pair on a coordinate plane. Draw a line that passes through the points. The line is the graph of the linear function.

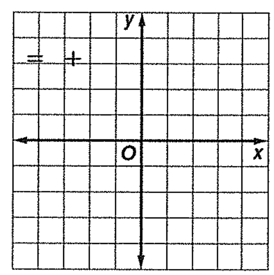


Exercises

Complete the function table. Then graph the function.

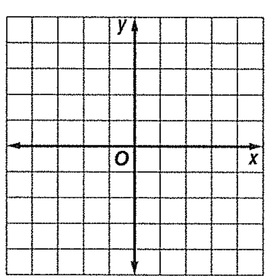
1. $y = x + 3$

x	$x + 3$	y	(x, y)
−2			
0			
1			
2			

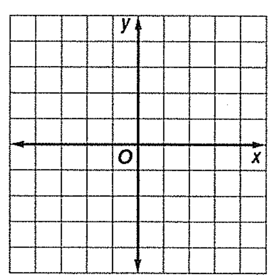


Graph each function.

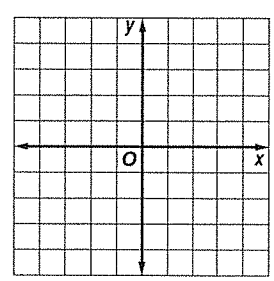
2. $y = 3x + 2$



3. $y = 2 - x$



4. $y = 3x - 1$



Determine whether each set of data is continuous or discrete.

5. the size of airmail packages

6. the number of boxes in an airmail shipment

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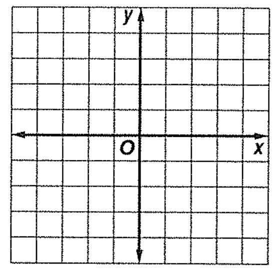
Skills Practice

Linear Functions

Complete the function table. Then graph the function.

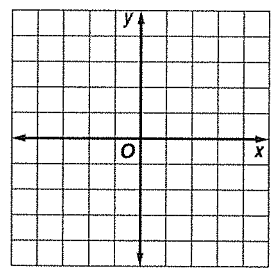
1. $y = x + 4$

x	$x + 4$	y	(x, y)
−2			
−1			
0			
1			



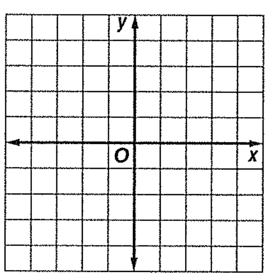
2. $y = 2x - 1$

x	$2x - 1$	y	(x, y)
−1			
0			
1			
2			

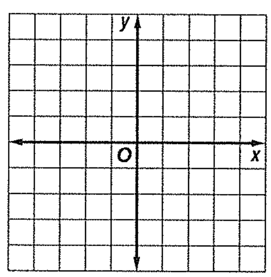


Graph each function.

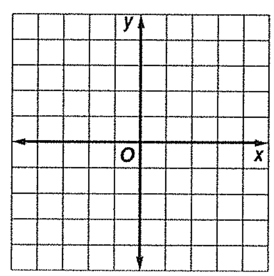
3. $y = x - 6$



4. $y = 2x - 3$



5. $y = 1 - x$



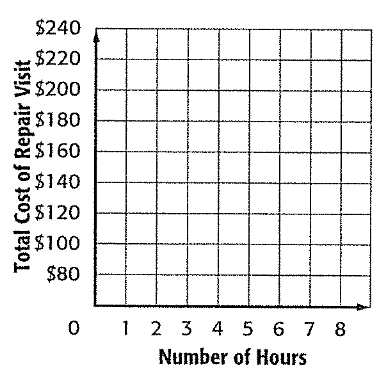
6. REPAIRS An appliance repairman charges $60 for a service call plus an additional $40 per hour to repair appliances.

a. Write a function to represent the situation.

b. Make a function table to find the total cost for 1, 2, 3, or 4 hours of work on an appliance.

x	1	2	3	4

c. Graph the function. Is the function continuous or discrete? Explain.



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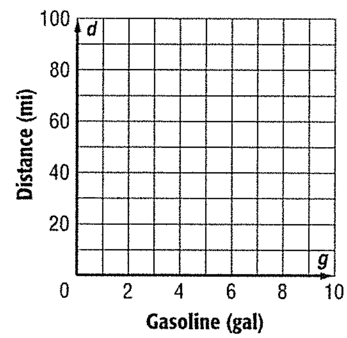
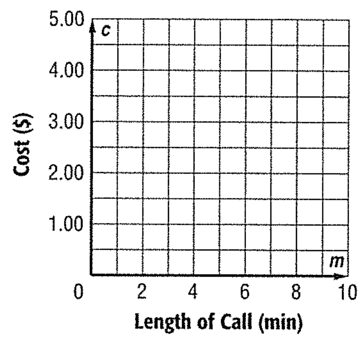
Problem-Solving Practice

Linear Functions

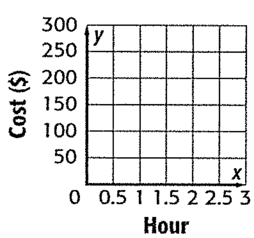
1. FUEL CONSUMPTION The function $d = 18g$ describes the distance d that Rick can drive his truck on g gallons of gasoline. Graph this function. Why is it sufficient to graph this function in the upper right quadrant only? How far can Rick drive on 2.5 gallons of gasoline?



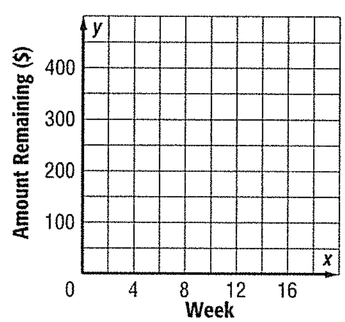
2. HOTELS The function $c = 0.5m + 1$ describes the cost c in dollars of a phone call that lasts m minutes made from a room at the Shady Tree Hotel. Graph the function. Use the graph to determine how much a 7-minute call will cost.



3. A computer store charges $45 for materials and $50 an hour for service to install two new programs and a connection. The cost $C(h)$ is a function of the number of hours h it takes to do the job. Graph the function $C(h) = 45 + 50h$. How much will a 3-hour installation cost?

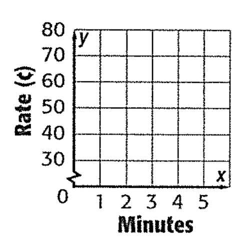


4. GIFTS Jonah received $300 in cash gifts for his fourteenth birthday. The function $y = 300 - 25x$ describes the amount y remaining after x weeks if Jonah spends $25 each week. Graph the function and determine the amount remaining after 9 weeks.



5. GIFTS Explain how you can use your graph in Exercise 4 to determine during which week the amount remaining will fall below $190. Then find the week.

6. Ron got a cell phone rate of $C(a) = 0.22 + 0.10a$. Graph the cost per minute. How much will a five-minute call cost?



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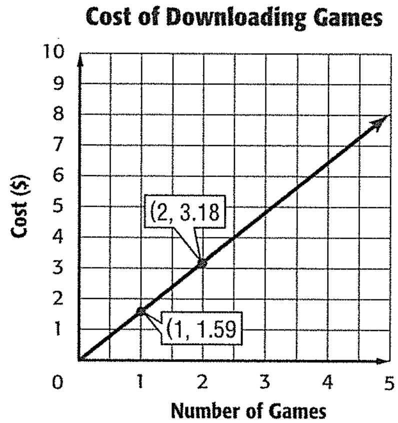
Skills Practice

Compare Properties of Functions

1. Cassie is downloading music and games onto her phone. It costs $0.99 to download a song to her phone. The costs of downloading games are shown in the graph. Compare the functions for each kind of download by comparing the costs.



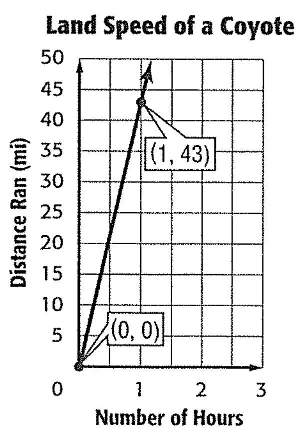
2. The number of gallons y a pool drains in x minutes is represented by the function $y = 20x$. The table shows the time it takes to fill up a pool. Compare the functions for each process by comparing the times.

Number of Minutes	Number of Gallons
1	15
2	30
3	45

3. The speeds of a coyote and giraffe are shown in the graph and table below.

a. Compare the functions by comparing the rates of change.

b. How much farther does a coyote run than a giraffe after 3 hours?



Land Speed of a Giraffe	
Number of Hours	**Distance Ran (mi)**
0.5	16
1	32
1.5	48

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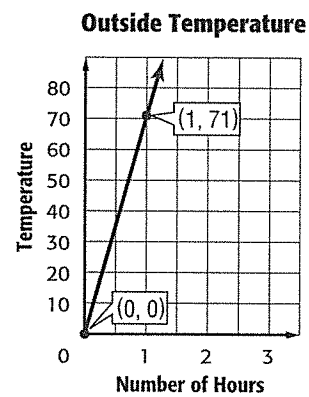
Problem-Solving Practice

Compare Properties of Functions

1. Anne kept track of the number of steps she took in a day using a pedometer. The average number of steps she took y per hour x can be represented by the function $y = 700x$. The table below shows the number of steps per hour that Elyse walked. Compare the functions for each person by comparing the number of steps.

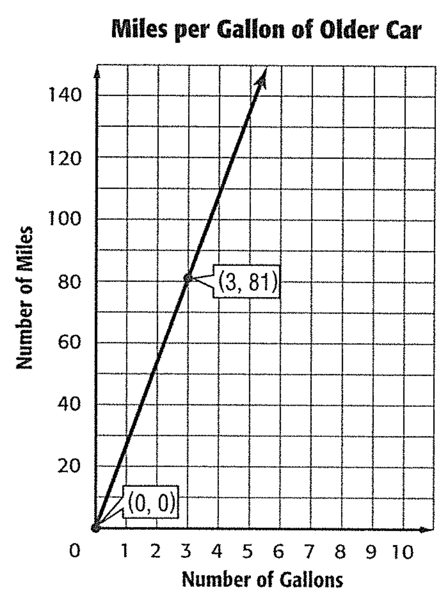
Number of Hours	Number of Steps
1	575
2	1,150
3	1,725

2. The graph shows the outside temperature after a number of hours. The inside temperature y after x hours can be represented by the function $y = 68x$. Compare the functions by comparing the temperatures.



3. For every computer that is sold, Kendall receives \$250 in commissions. The amount of commissions that Peter receives can be represented by the function $y = 225x$ where y is his commission and x is the number of computers sold. How much more does Kendall receive in commissions than Peter if they both sell 5 computers?

4. A new car gets 33 miles per gallon of gas. The graph shows the number of miles y that an older car gets per gallon x of gas. Compare the miles per gallon for each car.



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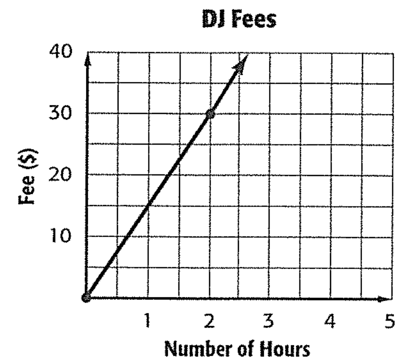
Reteach

Construct Functions

The *initial value of a function* is the corresponding *y*-value when *x* equals 0. You can find the initial value of a function from tables, graphs, and words.

Example 1

The Student Council is hiring a DJ for the school dance. The first DJ charges $25 an hour. The second DJ's fees areshown in the graph. Compare the functions for each DJ by comparing the fees.



Compare the rates of change.

The first DJ charges $25 an hour. To find the second DJ's fees, choose two points on the line and find the rate of change between them.

$$\frac{\text{Change in hours}}{\text{Change in fee}} = \frac{3-0}{2-0} \text{ or } 15$$

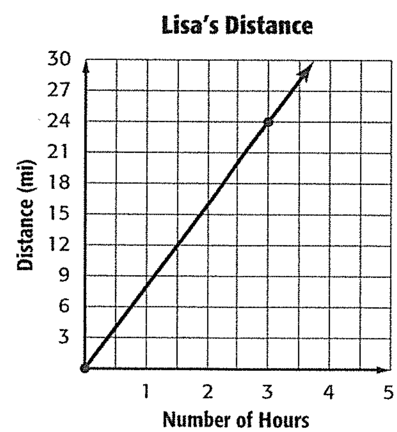
The second DJ charges $15 per hour. Since $15 < 25$, the second DJ charges less per hour than the first DJ.

Exercises

1. Melanie charges $7.50 an hour to babysit. The table shows how much Luisa charges for babysitting. Compare the functions by comparing their rates of change.

Number of Hours	Cost of Babysitting ($)
1	8
2	16
3	24

2. Tom and Lisa each spent an afternoon biking on neighborhood trails. The distance y Tom traveled can be represented by the function $y = 11x$. The graph shows Lisa's distance.



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