UNIT 4
LINEAR FUNCTIONS

UNIT 5
LINEAR MODELS & TABLES

WORTH COUNTY MIDDLE SCHOOL
8th Grade Mathematics
2015-2016

OVERVIEW

In this unit students will:

- graph proportional relationships;
- interpret unit rate as the slope;
- compare two different proportional relationships represented in different ways;
- use similar triangles to explain why the slope is the same between any two points on a non-vertical line;
- derive the equation $y = mx$ for a line through the origin;
- derive the equation $y = mx + b$ for a line intercepting the vertical axis at b; and
- interpret equations in $y = mx + b$ form as linear functions.

This unit focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade unit rates were expanded to complex fractions and percents through solving multi-step problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance time problems are notorious in mathematics. In this unit, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed, and described in different ways: graphically and algebraically. Students create representative graphs and the meaning of various points. They then compare the same information when represented in an equation.

By using coordinate grids and various sets of three similar triangles, students prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students generalize the slope to $y = mx$ for a line through the origin and $y = mx + b$ for a line through the vertical axis at b.

In Grade 8, the focus is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$. Students also need experiences with nonlinear functions, including functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule, such as a girl's height as a function of her age. Students learn that proportional relationships are part of a broader group of linear functions, and they are able to identify whether a relationship is linear. Nonlinear functions are included for comparison. Later, in high school, students use function notation and are able to identify types of nonlinear functions.

In the elementary grades, students explore number and shape patterns (sequences), and they use rules for finding the next term in the sequence. At this point, students describe sequences both by rules relating one term to the next and also by rules for finding the *n*th term directly. (In high school, students will call these recursive and explicit formulas.) Students express rules in both words and in symbols. Instruction focuses on additive and multiplicative sequences as well as sequences of square and cubic numbers, considered as areas and volumes of cubes, respectively. Students compute the area and perimeter of different-size squares and identify that one relationship is linear while the other is not by either looking at a table of value or a graph in which the side length is the independent variable (input) and the area or perimeter is the dependent variable (output).

When plotting points and drawing graphs, students develop the habit, based upon the context, of determining whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots can be misleading.

Students examine the graphs of linear functions and use graphing calculators or computer software to analyze or compare at least two functions at the same time. Illustrate with a slope triangle where the run is "1" that slope is the "unit rate of change." Compare this in order to compare two different situations and identify which is increasing/decreasing at a faster rate.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

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STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CONTENT STANDARDS

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

BIG IDEAS

- Patterns and relationships can be represented graphically, numerically, and symbolically.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- Functions are a special type of relationship that uniquely associates members of one set with members of another set.
- The understanding of functions is strengthened when they are explored across representations because each one provides a different view of the same relationship.

ESSENTIAL QUESTIONS

- How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?
- How can the same mathematical idea be represented in a different way? Why would that be useful?
- What is the significance of the patterns that exist between the triangles created on the graph of a linear function?
- When two functions share the same rate of change, what might be different/the same about their each of their representations? W
- What does the slope of the function line tell me about the unit rate?
- What does the unit rate tell me about the slope of the function line?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- determining unit rate
- applying proportional relationships
- recognizing a function in various forms
- plotting points on a coordinate plane
- understanding of writing rules for sequences and number patterns
- differences in graphing of discrete and continuous data
- attributes of similar figures

4

*FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- **flexibly use a combination of deep understanding, number sense, and memorization.**
- **are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.**
- **are able to articulate their reasoning.**
- **find solutions through a number of different paths.**

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety

OVERVIEW

In this unit students will:

- identify the rate of change and the initial value from tables, graphs, equations, or verbal descriptions;
- write a model for a linear function;
- sketch a graph when given a verbal description of a situation;
- analyze scatter plots;
- informally develop a line of best fit;
- use bivariate data to create graphs and linear models; and
- recognize patterns and interpret bivariate data.

Students are given opportunities and examples to figure out the meaning of $y = mx + b$. They will be able to "see" m and b in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts.

Using graphing calculators and web resources, students explore linear functions within context to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.

Students gather their own data or graphs in contexts they understand. Students measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar.

Students take a function in symbolic form and create a problem situation in words to match the function. Given a graph, students create a scenario that would fit the graph. Students sort a set of "cards" to match graphs, tables, equations, and problem situations and explain their reasoning to each other.

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. Students will extend their descriptions and understanding of variation to the graphical displays of bivariate data.

Scatter plots are the most common form of displaying bivariate data in Grade 8. Students practice informally finding the line of best fit using a scatter plot. Students create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students have a rich discussion about the effects of the change on the graph. Students use a graphing calculator to determine a linear regression and discuss how this relates to the graph. Students informally draw a line of best fit for a scatter plot and informally assess the fit of a function to their data.

The study of the line of best fit ties directly to the algebraic study of slope and intercept. Students interpret the slope and intercept of the line of best fit in the context of the data. Then students make predictions based on the line of best fit. Student will construct and interpret two-way tables in order to summarize two categorical variables.

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STANDARDS FOR MATHEMATICAL CONTENT

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. **Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.**
b. **Use relative frequencies calculated for rows or columns to describe possible association between the two variables.** ***For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?***

BIG IDEAS

- Collecting and examining data can sometimes help one discover patterns in the way in which two quantities vary.
- Changes in varying quantities are often related by patterns which, once discovered, can be used to predict outcomes and solve problems.

- Written descriptions, tables, graphs, and equations are useful in representing and investigating relationships between varying quantities.
- Different representations (written descriptions, tables, graphs, and equations) of the relationships between varying quantities may have different strengths and weaknesses.
- Linear functions may be used to represent and generalize real situations.
- Slope and y-intercept are keys to solving real problems involving linear relationships.

ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations involving linear relationships?
- How can the properties of lines help me to understand graphing linear functions?
- What can I infer from the data?
- How can functions be used to model real-world situations?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating – a table, a graph or an equation? Why?
- How can you construct and interpret two-way tables?
- How can I determine if there is an association between two given sets of data?
- How can I find the relative frequency using two-way tables?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- identifying and calculating slope
- identifying the y-intercept
- creating graphs using given data
- analyzing graphs
- making predictions from a graph

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SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the Common Core State Standards Mathematics Glossary and/or the Common Core GPS Mathematics Glossary when available.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **Model:** A mathematical representation of a process, device, or concept by means of a number of variables.
- **Interpret:**
- **Initial Value:** y-intercept.
- **Qualitative Variables:**
- **Linear:**
- **Non-linear:**
- **Slope:**
- **Rate of Change:**
- **Bivariate Data:** The following website has a short powerpoint (the 2nd one) that may be helpful. http://www.sophia.org/packets/bivariate-data-two-variables--2
- **Quantitative Variables:**
- **Scatter Plot:**
- **Line of Best Fit:**
- **Clustering:** The partitioning of a data set into subsets (clusters), so that the data in each subset (ideally) share some common trait - often similarity or proximity for some defined distance measure.
- **Outlier:**

FORMATIVE ASSESSMENT LESSONS (FAL)

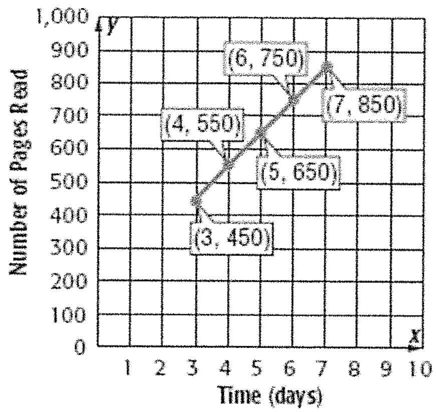
Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

Functions

Construct Functions
Page 323

A teacher read part of a book to a class. The graph shows the number of pages read by the teacher over the next several days. Find and interpret the rate of change and initial value.



To find the rate of change, choose two points from the graph.

$$\frac{\text{Change in number of pages}}{\text{Change in time}} = \frac{(550-450)\text{ pages}}{(4-3)\text{ days}}$$
$$= \frac{100\text{ pages}}{1\text{ day}}$$

The rate of change is 100. So, the teacher read 100 pages per day over the next several days.

Next, find the y-intercept. Extend the line so it intersects the y-axis. The value for y when $x = 0$ is 150. So, the teacher initially read 150 pages.

3 A teacher already had a certain number of canned goods for the food drive. Each day of the food drive, the class plans to bring in 10 cans. The total number of canned goods for day 10 is 205. Assume the relationship is linear. Find and interpret the rate of change and the initial value.

Since each week the class plans to bring in 10 cans, the rate of change is 10. To find the initial value, use the slope-intercept form to find the y-intercept. The point (10, 205) represents 205 cans on day 10.

$y = mx + b$	Slope-intercept form
$y = 10x + b$	Replace m with the rate of change, 10.
$205 = 10(10) + b$	Replace y with 205 and x with 10.
$105 = b$	Solve for b.

The y-intercept is 105. So, the teacher initially had 105 cans.

Writing Function Rules

Functions can be represented as tables, graphs, equations, physical models, or in words.

Mathematics Standards of Learning Curriculum Framework 2009: Grade 8, p. 25

Example 1: Tim's salary as a lifeguard depends on the number of hours he works. If he is paid $9.00 an hour, what is his salary for 3 hours? 12 hours? 22 hours?

Is his salary a function of the hours he works? Explain. ______________________

If possible, write the rule. Then, create a table of values.

Words: ______________________

Equation: ______________________

Input	Rule	Output

Example 2: The distance that Missy rides her bike depends on the number of minutes that she spends riding her bike. If she rides her bike at a constant rate of 0.15 miles per minute, what distance does Missy ride her bike in 15 minutes? 30 minutes? 1 hour?

Is the distance she rides her bike a function of the number of minutes she bikes? Explain. __________

If possible, write the rule. Then, create a table of values.

Words: ______________________

Equation: ______________________

Input	Rule	Output

Example 3: Is the cost a function of the number of items? Explain. ______________

Input (items)	4	9	1	8	4
Output (cost)	$6	$12	$2.50	$3	$15

If possible, write the rule.

Words: ______________________

Equation: ______________________

Example 4: Is the price a function of the number of donuts? Explain. ________________

Input (number of donuts)	1	2	3	4	5
Output (cost of the donuts)	\$1.25	\$2.50	\$3.75	\$5.00	\$6.25

If possible, write the rule.

Words: ______________________________________

Equation: ____________________________________

Example 5: Some plants need a large amount of space in order to grow. The number of seeds that can be planted in each row is related to the length of the row. Examine the table below. Does the relationship represent a function? Explain.

____

Input (row length)	Output (# of seeds per row)
16	4
24	6
52	13
64	16

If possible, write the rule.

Words: ______________________________________

Equation: ____________________________________

Example 6: Is the relationship a function? Explain. If possible, write the function rule.

a.

Input	2	5	0	6	4
Output	4	25	0	36	16

____

____

b.

Input	1	2	3	2	1
Output	10	20	30	40	50

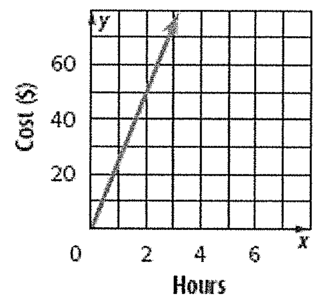
____

____

Equations in Two Variables

Equations in $y = mx$ Form
Pages 195–196

3 **Tom was comparing computer repair companies. The cost y for Computer Access for x hours is shown in the graph. The cost for Computers R Us can be represented by the equation $y = 23.5x$. Which company's repair price is lower? Explain.**



Compare the rates of change.

Computers R Us: $y = 23.5x$; The unit rate is \$23.50 per hour.

Computer Access: The slope of the graph is $\frac{50}{2}$ or \$25 per hour.

Since $23.5 < 25$, Computers R Us has the lower repair price.

7 **The number of centimeters varies directly with the number of inches. Find the measure of an object in centimeters if it is 50 inches long.**

Inches, x	6	9	12	15
Centimeters, y	15.24	22.86	30.48	38.10

Write an equation of direct variation. Let x represent the measure of the object in inches and let y represent the measure of the object in centimeters.

$y = mx$	Write the equation.
$15.24 = m(6)$	$y = 15.24$, $x = 6$
$2.54 = m$	Simplify.
$y = 2.54x$	Replace m with 2.54

Use the equation to find y when $x = 50$.

$y = 2.54x$	Write the equation.
$y = 2.54(50)$	$x = 50$
$y = 127$	Simplify.

So, the object is 127 centimeters long.

NAME ______________________ DATE ____________ PERIOD ________

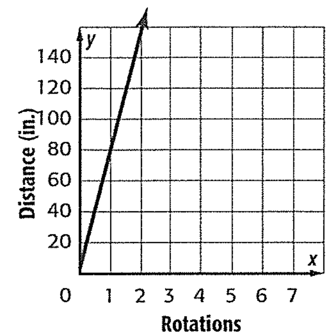
Reteach

Equations in y = mx Form

When the ratio of two variable quantities is constant, their relationship is called a **direct variation**.

Example 1

The distance that a bicycle travels varies directly with the number of rotations that its tires make. Determine the distance that the bicycle travels for each rotation.



Since the graph of the data forms a line, the rate of change is constant. Use the graph to find the constant ratio.

$\frac{\text{distance traveled}}{\text{\# of rotations}} \longrightarrow \frac{80}{1} \quad \frac{160}{2} \text{ or } \frac{80}{1} \quad \frac{240}{3} \text{ or } \frac{80}{1} \quad \frac{320}{4} \text{ or } \frac{80}{1}$

The bicycle travels 80 inches for each rotation of the tires.

Example 2

The number of trading cards varies directly as the number of packages. If there are 84 cards in 7 packages, how many cards are in 12 packages?

Let x = the number of packages and y = the total number of cards.

$y = mx$	Direct variation equation
$84 = m(7)$	$y = 84, x = 7$
$12 = m$	Simplify.
$y = 12x$	Substitute for $m = 12$.

Use the equation to find y when $x = 12$.

$y = 12x$	
$y = 12(12)$	$x = 12$
$y = 144$	Multiply.

There are 144 cards in 12 packages.

Exercises

Write an equation and solve the given situation.

1. TICKETS Four friends bought movie tickets for \$41. The next day seven friends bought movie tickets for \$71.75. What is the price of one ticket?

2. JOBS Barney earns \$24.75 in three hours. If the amount that he earns varies directly with the number of hours, how much would he earn in 20 hours?

NAME ______________________ DATE ____________ PERIOD ________

Skills Practice

Equations in y = mx Form

For Exercises 1–3, determine whether each linear function is a direct variation. If so, state the constant of variation.

1.

Price, x	\$5	\$10	\$15	\$20
Tax, y	\$0.41	\$0.82	\$1.23	\$1.64

2.

Hours, x	11	12	13	14
Distance, y (miles)	154	167	180	193

3.

Age, x	8	9	10	11
Grade, y	3	4	5	6

For Exercises 4–12, y varies directly with x. Write an equation for the direct variation. Then find each value.

4. If $y = 8$ when $x = 3$, find y when $x = 45$.

5. If $y = -4$ when $x = 10$, find y when $x = 2$.

6. If $y = 27$ when $x = 8$, find y when $x = 11$.

7. Find y when $x = 12$, if $y = 2$ when $x = 5$.

8. Find y when $x = 3$, if $y = -4$ when $x = -9$.

9. Find y when $x = -6$, if $y = 15$ when $x = -5$.

10. If $y = 20$ when $x = 8$, what is the value of x when $y = -2$?

11. If $y = -30$ when $x = 15$, what is the value of x when $y = 60$?

12. If $y = 42$ when $x = 15$, what is the value of x when $y = 70$?

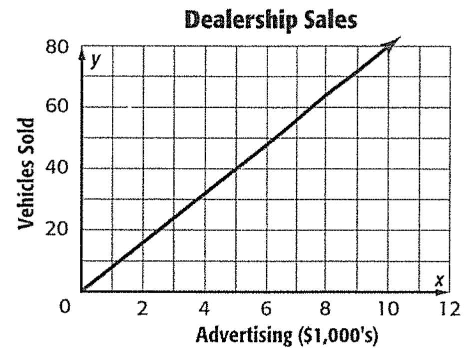
18

NAME ______________________ DATE ____________ PERIOD ________

Homework Practice

Equations in y = mx Form

1. ADVERTISING The number of vehicles a dealership sells varies directly with the money spent on advertising. How many vehicles does the dealership sell for each $1,000 spent on advertising?



2. SNOWMOBILES Bruce rents snowmobiles to tourists. He charges $135 for 4 hours and $202.50 for 6 hours. What is the hourly rate Bruce charges to rent a snowmobile?

3. SOLAR ENERGY The power absorbed by a solar panel varies directly with its area. If an 8 square meter panel absorbs 8,160 watts of power, how much power does a 12 square meter solar panel absorb?

4. INSECT CONTROL Mr. Malone used 40 pounds of insecticide to cover 1,760 square feet of lawn and 60 pounds to cover an additional 2,640 square feet. How many pounds of insecticide would Mr. Malone need to cover his whole lawn of 4,480 square feet?

Determine whether each linear function is a direct variation. If so, state the constant of variation.

5.

Volume, x	2	4	6	8
Mass, y	10	20	30	40

6.

Gallons, x	5	10	15	20
Miles, y	95	190	285	380

7.

Time, x	8	9	10	11
Temp, y	68	71	74	77

8.

Age, x	3	6	9	12
Height, y	28	40	52	64

ALGEBRA **If y varies directly with x, write an equation for the direct variation. Then find each value.**

9. If $y = -5$ when $x = 2$, find y when $x = 8$.

10. Find y when $x = 1$, if $y = 3$ when $x = 2$.

11. If $y = -7$ when $x = -21$, what is the value of x when $y = 9$?

12. Find x when $y = 18$, if $y = 5$ when $x = 4$.

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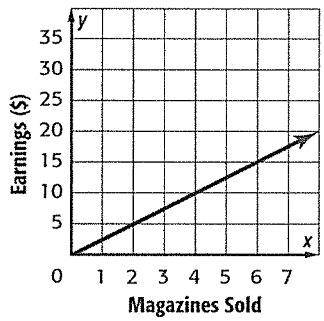
NAME ______________________ DATE ____________ PERIOD ________

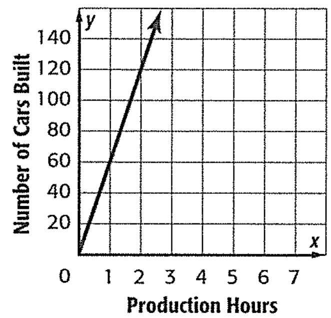
Problem-Solving Practice

Equations in y = mx Form

1. JOBS The amount Candice earns varies directly with the number of magazines she sells. How much does Candice earn for each magazine sale?



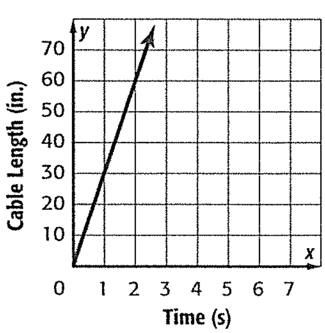
2. MANUFACTURING The number of cars built varies directly as the number of hours the production line operates. What is the ratio of cars built to hours of production?



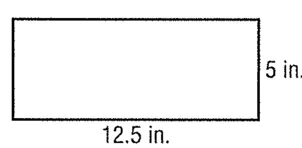
3. DRIVING A car drives 283.5 miles in 4.5 hours. Assuming that the distance traveled varies directly with the time traveled, how far will the car travel in 7 hours?

4. MEASUREMENT The number of kilograms that an object weighs varies directly as the number of pounds. If an object that weighs 45 kilograms weighs about 100 pounds, about how many kilograms is an object that weighs 70 pounds?

5. RECORDING The amount of cable that is wound on a spool varies directly with the amount of time that passes. Determine the speed at which the cable moves.



6. GEOMETRY The width of a rectangle varies directly as its length. What is the perimeter of a rectangle that is 15 inches long?



Equations in Two Variables

Slope-Intercept Form
Page 203

5 **Write an equation of a line in slope-intercept form with a slope of $\frac{5}{6}$ and a y-intercept of 8.**

$y = mx + b$ Slope-intercept form

$y = \frac{5}{6}x + 8$ Replace m with $\frac{5}{6}$ and b with 8.

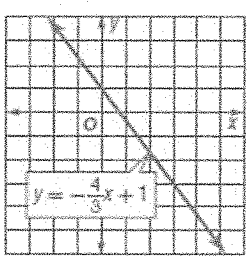
11 **Graph the equation $y = -\frac{4}{3}x + 1$.**

Find the slope and y-intercept.

$y = -\frac{4}{3}x + 1$ slope $= -\frac{4}{3}$, y-intercept $= 1$

Graph the y-intercept at (0, 1).

Use the slope to locate a second point on the line. Go down 4 units and right 3 units. Then draw a line through the points.



NAME ______________________ DATE ____________ PERIOD ________

Reteach

Slope-Intercept Form

Linear equations are often written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y-intercept.

Example 1

State the slope and the y-intercept of the graph of $y = x - 3$.

$y = x - 3$	Write the original equation.
$y = 1x + (-3)$	Write the equation in the form $y = mx + b$.
$y = mx + b$	$m = 1, b = -3$

The slope of the graph is 1, and the y-intercept is -3.

You can use the slope-intercept form of an equation to graph the equation.

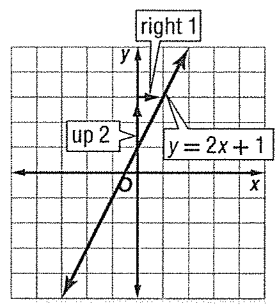
Example 2

Graph $y = 2x + 1$ using the slope and y-intercept.

Step 1 Find the slope and y-intercept.
$y = 2x + 1$ slope = 2, y-intercept = 1

Step 2 Graph the y-intercept 1.

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.
$m = \frac{2}{1}$ ← change in y : up 2 units; ← change in x : right 1 unit



Step 4 Draw a line through the two points.

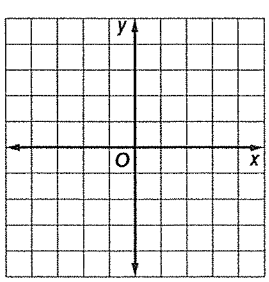
Exercises

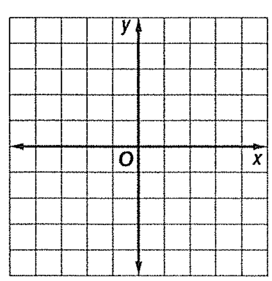
State the slope and the y-intercept for the graph of each equation.

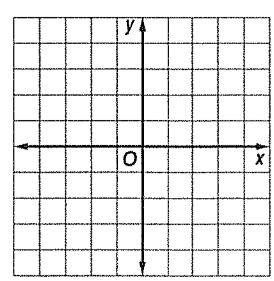
1. $y = x + 1$ **2.** $y = 2x - 4$ **3.** $y = \frac{1}{2}x - 1$

Graph each equation using the slope and the y-intercept.

4. $y = 2x + 2$ **5.** $y = x - 1$ **6.** $y = \frac{1}{2}x + 2$







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Skills Practice

Slope-Intercept Form

State the slope and the y-intercept for the graph of each equation.

1. $y = x + 4$

2. $y = 2x - 2$

3. $y = 3x - 1$

4. $y = -x + 3$

5. $y = \frac{1}{2}x - 5$

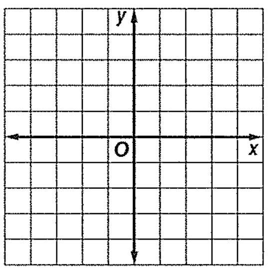
6. $y = -\frac{1}{3}x + 4$

7. $y - 2x = -1$

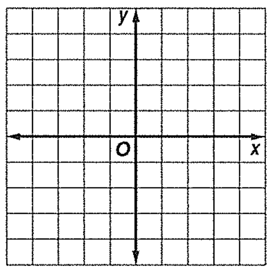
8. $y + 4x = 2$

9. $y = \frac{3}{2}x - 3$

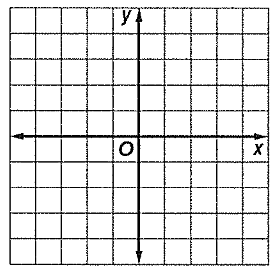
10. Graph a line with a slope of 1 and a y-intercept of -4.



11. Graph a line with a slope of 2 and a y-intercept of -3.

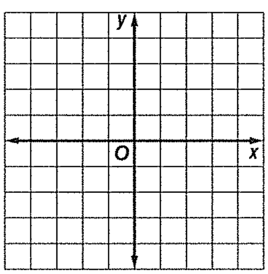


12. Graph a line with a slope of $\frac{1}{3}$ and a y-intercept of 1.

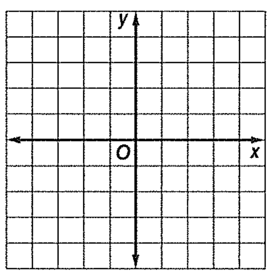


Graph each equation using the slope and the y-intercept.

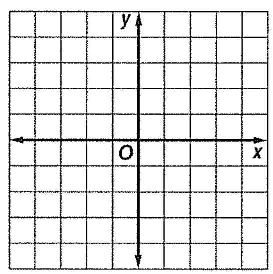
13. $y = 3x - 3$



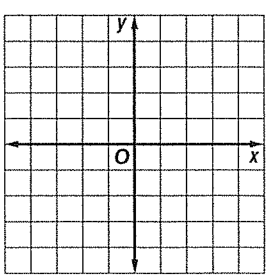
14. $y = -x + 1$



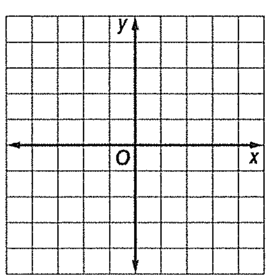
15. $y = \frac{1}{2}x - 2$



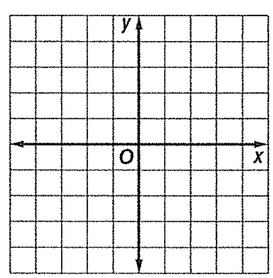
16. $y = 4x - 2$



17. $y = -\frac{3}{2}x + 1$



18. $y = \frac{2}{3}x - 3$



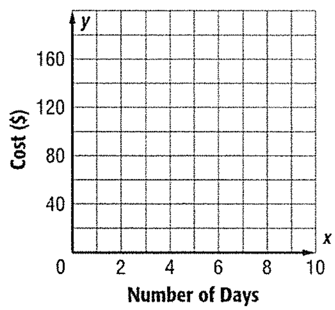
NAME ______________________ DATE ____________ PERIOD ______

Problem-Solving Practice

Slope-Intercept Form

CAR RENTAL For Exercises 1 and 2, use the following information. Ace Car Rentals charges $20 per day plus a $10 service charge to rent one of its compact cars. The total cost can be represented by the equation $y = 20x + 10$, where x is the number of days and y is the total cost.

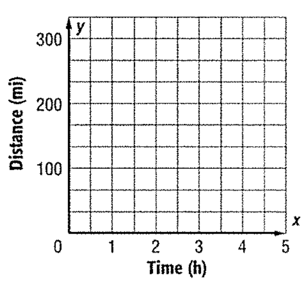
1. Graph the equation. What do the slope and y-intercept represent?



2. Explain how to use your graph to find the total cost of renting a compact car for 7 days. Then find this cost.

TRAVEL For Exercises 3 and 4, use the following information. Thomas is driving from Oak Ridge to Lakeview, a distance of 300 miles. He drives at a constant 60 miles per hour. The equation for the distance yet to go is $y = 300 - 60x$, where x is the number of hours since he left.

3. What is the slope and y-intercept? Explain how to use the slope and y-intercept to graph the equation. Then graph the equation.



4. Explain how to find the total travel time. Then find this time.

5. WEATHER The equation $y = 0.2x + 3.5$ can be used to find the amount of accumulated snow y in inches x hours after 5 P.M. on a certain day. Identify the slope and y-intercept of the graph of the equation and explain what each represents.

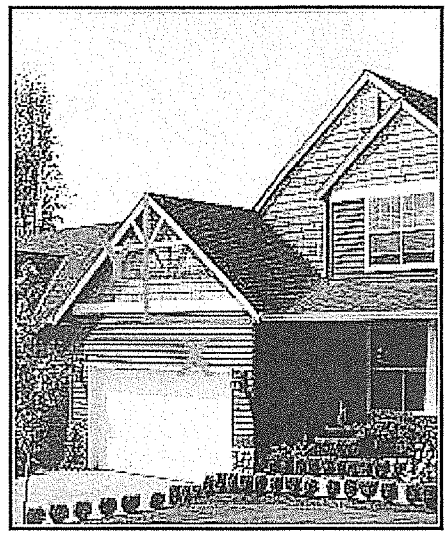
6. SALARY Janette's weekly salary can be represented by the equation $y = 500 + 0.4x$, where x is the dollar total of her sales for the week. Identify the slope and y-intercept of the graph of the equation and explain what each represents.

Slope and Rate of Change

The word **slope** (gradient, incline, pitch) is used to describe the measurement of the steepness of a straight line. The higher the slope, the steeper the line. The slope of a line is a *rate of change.*

$$Slope = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Rise}}{\text{Run}}$$

The building code for using asphalt shingles on roofs states that the minimum pitch must be a rise of 4" for every 12" of horizontal distance (run) covered. Asphalt shingles are not to be used on roofs that have very little steepness. Builders check to see if the pitch (slope) of the roof is $\frac{4}{12}$ or 4:12 or 4 to 12 before using asphalt shingles.



Builders need to know the pitch of a roof to determine which type of shingle will be appropriate for the roof.

Slope is a ratio and can be expressed as:						
change in y over change in x.	or	$\frac{\text{vertical change}}{\text{horizontal change}}$	or	$\frac{y_2 - y_1}{x_2 - x_1}$	or	$\frac{rise}{run}$

Let's examine the slope of a straight line more carefully.

Consider the line $y = 2x + 1$, shown at the right.

By how much has the value of y changed between the two points (-4,-7) and (-3,-5)? This will be a vertical change. Answer: 2 units

(as read from the left point to the right point, with the right point being "higher" on the graph)

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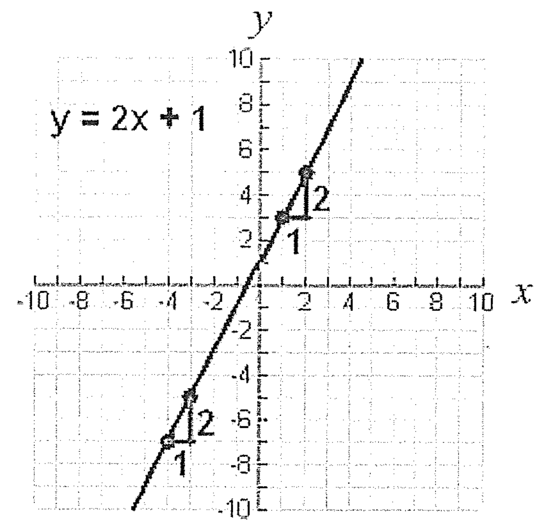
two points (-4,-7) and (-3,-5)? This will be a horizontal change. Answer: 1 unit (as read from the left point to the right point on the graph)

The slope $= \dfrac{\textit{vertical change}}{\textit{horizontal change}} = \dfrac{2}{1} = 2$

Notice that this slope will be the same if the points (1,3) and (2, 5) are used for the calculations. For straight lines, the rate of change (slope) is constant (always the same).



For every one unit that is moved on the x-axis, two units are moved on the y-axis. This is true at any location on the line.

Example:

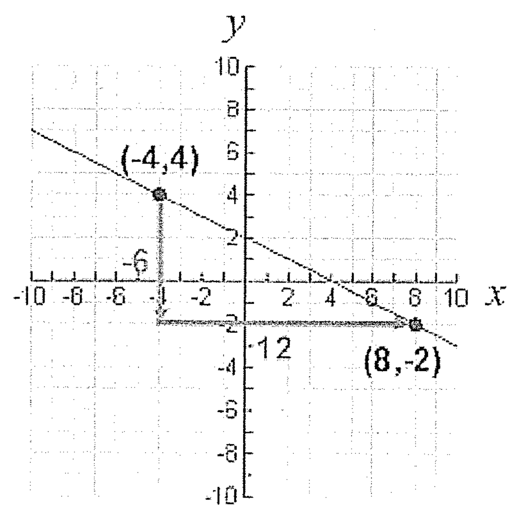
Find the slope of a line passing through the points (-4,4) and (8,-2).

You have several choices:

1. You can graph the points and "count" the vertical changes and horizontal changes to use in the formula:

$$\textit{Slope} = \frac{\textit{Vertical change}}{\textit{Horizontal change}} = \frac{\textit{Rise}}{\textit{Run}}$$

$$\textit{Slope} = \frac{-6}{12} = \frac{-1}{2}$$



Notice that to read the rise and run for these two points, we started at (-4,4), moved "down" (negative) 6 units and moved "right" (positive) 12 units.

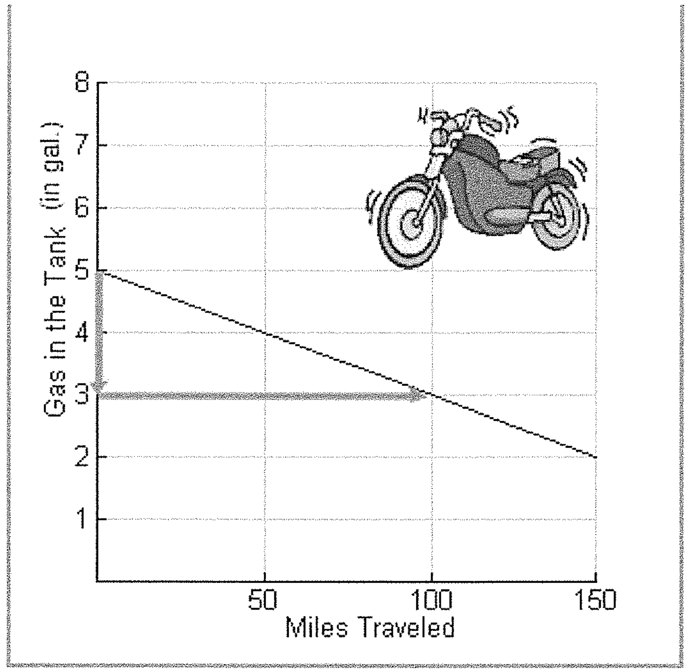
2. You can substitute the points directly into the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{-4-8} = \frac{6}{-12} = -\frac{1}{2}$$

Example:

Todd had 5 gallons of gasoline in his

motorbike. After driving 100 miles, he had 3 gallons left. The graph at the right shows Todd's situation.



a. Find the slope of the line.

$$Slope = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{-2}{100} = \frac{-1}{50}$$

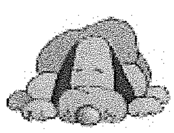
b. What does this slope tell us?

Since $\frac{-1}{50} = -0.02$, we know that Todd's bike is burning .02 gallons of gasoline for every mile that he travels. The negative value of the slope tells us that the amount of gasoline in the tank is decreasing.

c. What is Todd's mpg?

The $\frac{-1}{50} = \frac{\text{change in gallons}}{\text{change in miles}}$ tells us that Todd can drive 50 miles on one gallon of gasoline (an mpg of 50 miles per gallon).

While slopes of lines are not labeled with units, rates of change used in application problems often take on the units depicted in the problem, such as Todd's bike burning .02 gallons per mile, or his mpg being 50 miles per gallon.



NAME ______________________________ DATE ____________ PERIOD ______

Reteach

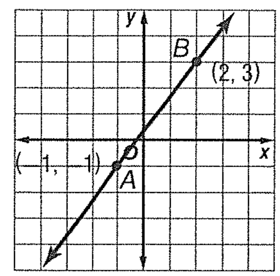
Slope

The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates. As an equation, the slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

Example 1

Find the slope of the line that passes through $A(-1, -1)$ and $B(2, 3)$.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

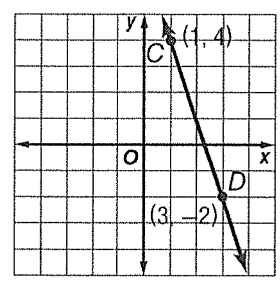
$m = \frac{3 - (-1)}{2 - (-1)}$ $(x_1, y_1) = (-1, -1)$, $(x_2, y_2) = (2, 3)$

$m = \frac{4}{3}$ Simplify.

Check When going from left to right, the graph of the line slants upward. This is correct for a positive slope.

Example 2

Find the slope of the line that passes through $C(1, 4)$ and $D(3, -2)$.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{-2 - 4}{3 - 1}$ $(x_1, y_1) = (1, 4)$, $(x_2, y_2) = (3, -2)$

$m = \frac{-6}{2}$ or -3 Simplify.

Check When going from left to right, the graph of the line slants downward. This is correct for a negative slope.

Exercises

Find the slope of the line that passes through each pair of points.

1. $A(0, 1), B(3, 4)$ **2.** $C(1, -2), D(3, 2)$ **3.** $E(4, -4), F(2, 2)$

4. $G(3, 1), H(6, 3)$ **5.** $I(4, 3), J(2, 4)$ **6.** $K(-4, 4), L(5, 4)$

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NAME ______________________ DATE ____________ PERIOD _______

Skills Practice

Slope

Find the slope of the line that passes through each pair of points.

1. $A(-2, -4)$, $B(2, 4)$

2. $C(0, 2)$, $D(-2, 0)$

3. $E(3, 4)$, $F(4, -2)$

4. $G(-3, -1)$, $H(-2, -2)$

5. $I(0, 6)$, $J(-1, 1)$

6. $K(0, -2)$, $L(2, 4)$

7. $O(1, -3)$, $P(2, 5)$

8. $Q(1, 0)$, $R(3, 0)$

9. $S(0, 4)$, $T(1, 0)$

10. $U(1, 3)$, $V(1, 5)$

11. $W(2, -2)$, $X(-1, 1)$

12. $Y(-5, 0)$, $Z(-2, -4)$

13. $A(2, -1)$, $B(-4, -4)$

14. $C(-2, 2)$, $D(-4, 2)$

15. $E(-1, -4)$, $F(-3, 0)$

16. $G(7, 4)$, $H(2, 0)$

17. $K(2, -2)$, $L(2, -3)$

18. $M(-1, -1)$, $N(-4, -5)$

19. $O(5, -3)$, $P(-3, 4)$

20. $Q(-1, -3)$, $R(1, 2)$

21. $W(3, 25)$, $X(1, 1)$

22. $Y(2, 2)$, $Z(-5, -4)$

23. $C(0, -2)$, $D(3, -2)$

24. $G(-3, 5)$, $H(-3, 2)$

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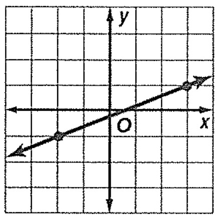
Homework Practice

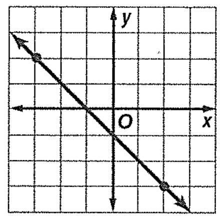
Slope

Find the slope of each line.

1.



2.



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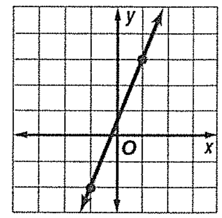
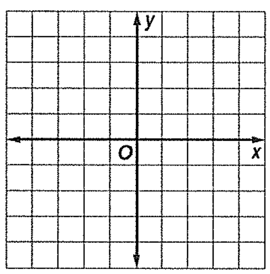


The points given in each table lie on a line. Find the slope of the line. Then graph the line.

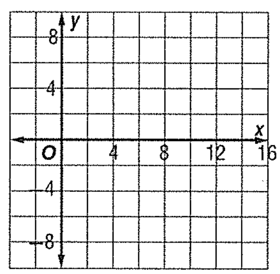
4.

x	−1	1	3	5
y	−2	0	2	4



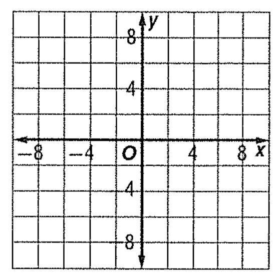
5.

x	−2	3	8	13
y	−2	−1	0	1

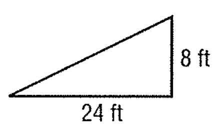


6.

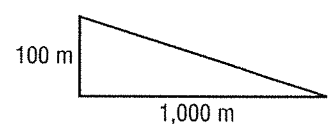
x	−1	2	5	8
y	3	−1	−5	9



7. HOMES Find the slope of the roof of a home that rises 8 feet for every horizontal change of 24 feet.



8. MOUNTAINS Find the slope of a mountain that descends 100 meters for every horizontal distance of 1,000 meters.



Find the slope of the line that passes through each pair of points.

9. $A(1, 3), B(4, 7)$

10. $C(3, 5), D(2, 6)$

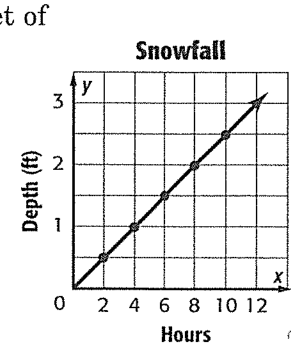
11. $E(4, 0), F(5, 5)$

12. SNOWFALL Use the graph at the right. It shows the depth in feet of snow after each two-hour period during a snowstorm.

a. Find the slope of the line.

b. Does the graph show a constant rate of change? Explain.

c. If the graph is extended to the right, could you expect the slope to remain constant? Explain.



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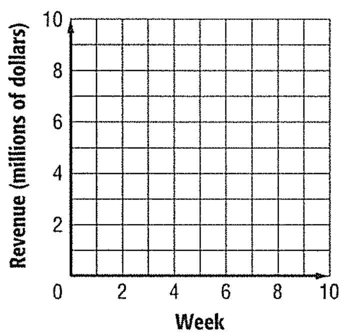
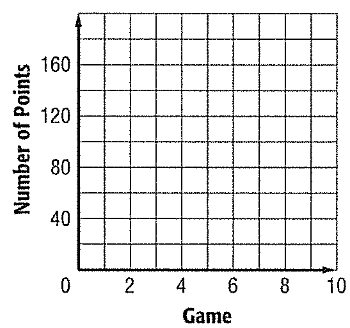
Problem-Solving Practice

Slope

1. MOVIES By the end of its first week, a movie had grossed \$2.3 million. By the end of its sixth week, it had grossed \$6.8 million. Graph the data with the week on the horizontal axis and the revenue on the vertical axis, and draw a line through the points. Then find and interpret the slope of the line.

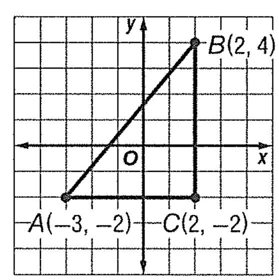


2. BASKETBALL After Game 1, Felicia had scored 14 points. After Game 5, she had scored a total of 82 points for the season. After Game 10, she had scored 129 points. Graph the data with the game number on the horizontal axis and the number of points on the vertical axis. Connect the points using two different line segments.



3. BASKETBALL Find the slope of each line segment in your graph from Exercise 2 and interpret it. Which part of the graph shows the greater rate of change? Explain.

4. GEOMETRY The figure shows triangle ABC plotted on a coordinate plane. Explain how to find the slope of the line through points A and B. Then find the slope.



5. Use the figure in Exercise 4. What is the slope of the line through points A and C? How do you know?

6. Use the figure in Exercise 4. What is the slope of the line through points B and C? How do you know?

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Enrich

Find the Code

Secret codes have been used for centuries to pass messages along without the meaning of the message being discovered. Many secret codes use mathematics and mathematical ideas. By finding the slopes given below, you will discover a secret message.

__________ (3, 2) and (5, 6)

__________ (−3, 3) and (−1, 2)

__________ (−1, 5) and (0, 4)

__________ (4, 7) and (−1, 4)

__________ (1, 6) and (2, 3)

__________ (−4, −6) and (−3, −1)

__________ (−2, 4) and (−1, 5)

__________ (−1, 5) and (0, 5)

__________ (0, 0) and (3, 2)

KEY:

A	B	C	D	E	F	G	H
$-\frac{1}{2}$	-2	3	6	-10	1	$-\frac{2}{3}$	$\frac{3}{5}$
I	J	K	L	M	N	O	P
-3	$-\frac{4}{5}$	$\frac{4}{5}$	$\frac{5}{6}$	2	$\frac{2}{3}$	-6	$\frac{9}{10}$
Q	R	S	T	U	V	W	X
$-\frac{7}{8}$	7	5	-1	0	-4	-9	$\frac{4}{7}$
Y	Z						
$\frac{1}{10}$	4						

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Extra Practice

Slope

Find the slope of the line that passes through each pair of points.

1. $A(2, 3), B(1, 5)$

2. $C(-6, 1), D(2, 1)$

3. $E(3, 0), F(5, 0)$

4. $G(-1, -3), H(-2, -5)$

5. $I(6, 7), J(11, 1)$

6. $K(5, 3), L(5, -2)$

7. $M(10, 2), N(-3, 5)$

8. $O(6, 2), P(1, 7)$

9. $Q(5, 8), R(-3, -2)$

10. $S(-1, 7), T(3, 8)$

11. $U(4, -1), V(-5, -2)$

12. $W(3, -2), X(7, -1)$

13. $Y(0, 5), Z(2, 1)$

14. $A(6, 5), B(-3, -5)$

15. $C(2, 1), D(7, -1)$

Congruence and Similarity

Slope and Similar Triangles
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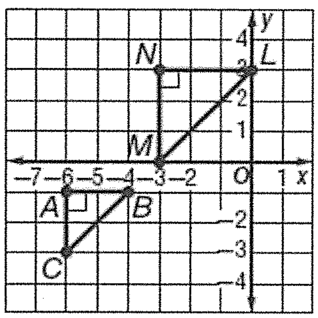
1 Graph triangle *ABC* with vertices *A*(−6, −1), *B*(−4, −1), and *C*(−6, −3) and triangle *NLM* with vertices *N*(−3, 3), *L*(0, 3), and *M*(−3, 0). Then write a proportion comparing the rise to the run for the similar slope triangles and find the numeric value.

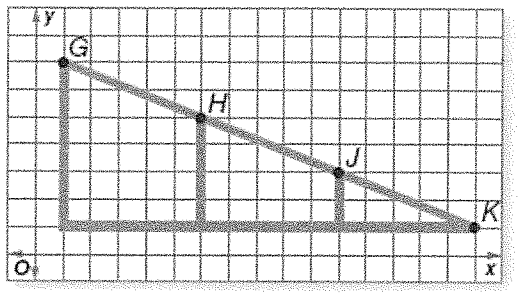
Graph the triangles. Then write the proportion.



$\frac{AC}{NM} = \frac{AB}{NL}$	Corresponding sides of similar triangles are proportional.
$AC \cdot NL = NM \cdot AB$	Find the cross products.
$\frac{AC \cdot NL}{AB \cdot NL} = \frac{NM \cdot AB}{AB \cdot NL}$	Division Property of Equality
$\frac{AC}{AB} = \frac{NM}{NL}$	Simplify.
$\frac{2}{2} = \frac{3}{3}$	$AC = 2, AB = 2, NM = 3, NL = 3$

Since $\frac{AC}{AB} = \frac{NM}{NL}$ or $\frac{2}{2} = \frac{3}{3}$, the numeric value is $\frac{1}{1}$.

3 **The plans for a skateboard ramp are shown. Use two points to find the slope of the ramp. Then verify that the slope is the same at a different location by choosing a different set of points.**



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Formula for slope

$m = \frac{5-7}{6-1}$ Use the points G and H. $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (6, 5)$.

$m = \frac{-2}{5}$ or $-\frac{2}{5}$ Simplify.

The slope of the ramp is $-\frac{2}{5}$. Verify that the slope is the same using two other points.

Sample answer:

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Formula for slope

$m = \frac{1-3}{16-11}$ Use the points J and K. $(x_1, y_1) = (11, 3)$ and $(x_2, y_2) = (16, 1)$.

$m = \frac{-2}{5}$ or $-\frac{2}{5}$ Simplify. The slope of the ramp is the same.

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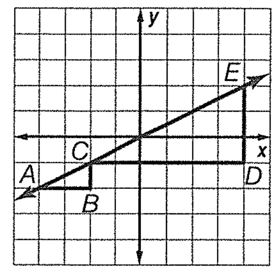
Reteach

Slope and Similar Triangles

Recall that the slope of a line is the ratio of the rise to the run. You can use properties of similar triangles to show the ratios of the rise to the run for each right triangle are equal.

Example

Write a proportion comparing the rise to the run for each of the similar slope triangles shown at the right. Then find the numeric value.



$\frac{CB}{ED} = \frac{BA}{DC}$ Corresponding sides of similar triangles are proportional.

$CB \cdot DC = ED \cdot BA$ Find the cross products.

$\frac{CB \cdot DC}{BA \cdot DC} = \frac{ED \cdot BA}{BA \cdot DC}$ Division Property of Equality

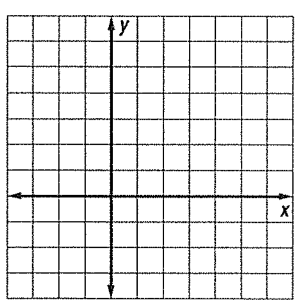
$\frac{CB}{BA} = \frac{ED}{DC}$ Simplify.

$\frac{1}{2} = \frac{3}{6}$ $CB = 1, BA = 2, ED = 3, DC = 6$

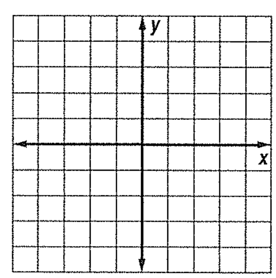
So, $\frac{CB}{BA} = \frac{ED}{DC}$, or $\frac{1}{2} = \frac{3}{6}$.

Exercises

1. Graph $\triangle XYZ$ with vertices $X(-3, 5)$, $Y(-3, 3)$, and $Z(0, 3)$ and $\triangle ZLP$ with vertices $Z(0, 3)$, $L(0, -1)$, and $P(6, -1)$. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.



2. Graph $\triangle ABE$ with vertices $A(-4, -3)$, $B(0, 0)$, and $E(0, -3)$ and $\triangle ACD$ with vertices $A(-4, -3)$, $C(4, 3)$, and $D(4, -3)$. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.



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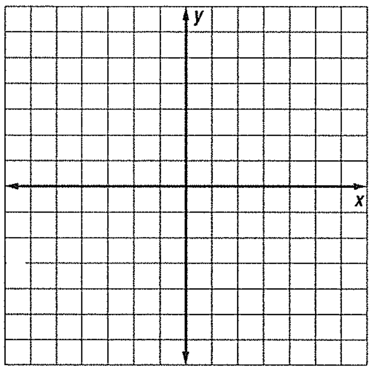
Skills Practice

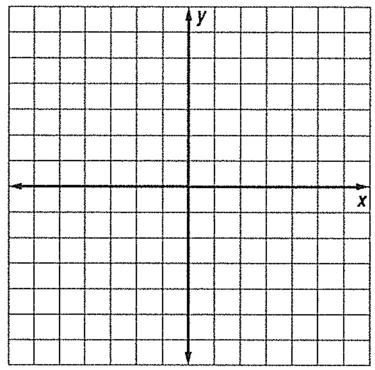
Slope and Similar Triangles

Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

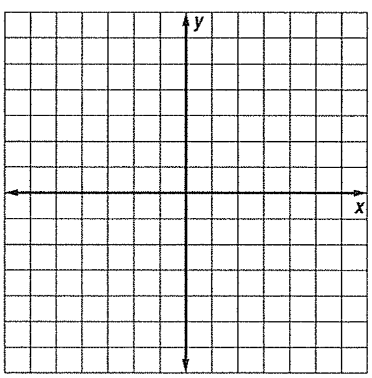
1. $\triangle CDE$ with vertices $C(-6, -3)$, $D(-3, -2)$, and $E(-3, -3)$; $\triangle MNO$ with vertices $M(0, -1)$, $N(6, 1)$, and $O(6, -1)$



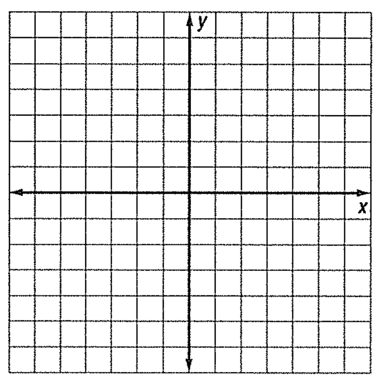
2. $\triangle RST$ with vertices $R(-4, 5)$, $S(-4, -4)$, and $T(2, -4)$; $\triangle UVW$ with vertices $U(-2, 2)$, $V(-2, -1)$, and $W(0, -1)$



3. $\triangle QRP$ with vertices $Q(-5, 1)$, $R(-1, 3)$, and $P(-1, 1)$; $\triangle RKJ$ with vertices $R(-1, 3)$, $K(5, 6)$, and $J(5, 3)$.



4. $\triangle CAM$ with vertices at $C(-1, 6)$, $A(-1, 3)$, and $M(0, 3)$; $\triangle CEN$ with vertices at $C(-1, 6)$, $E(-1, -3)$, and $N(2, -3)$



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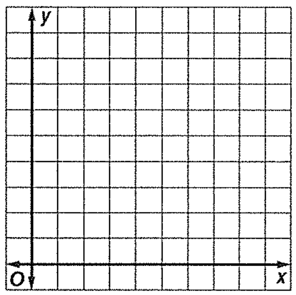
Homework Practice

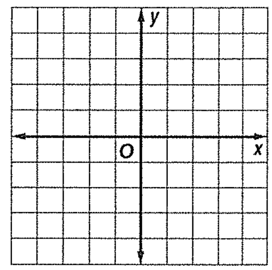
Slope and Similar Triangles

Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

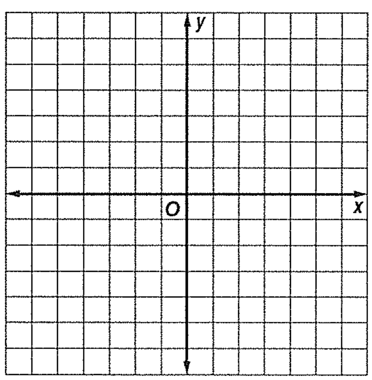
1. $\triangle EFG$ with vertices $E(1,9)$, $F(1,5)$, and $G(2,5)$; $\triangle GHI$ with vertices $G(2,5)$, $H(1,2)$, and $I(3,1)$



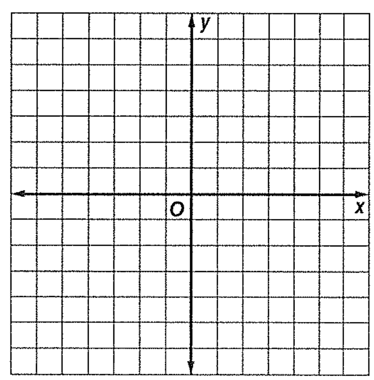
2. $\triangle JNL$ with vertices $J(-3,3)$, $N(-3,-3)$, and $L(5,-3)$; $\triangle KML$ with vertices $K(1,0)$, $M(1,-3)$, and $L(5,-3)$



3. $\triangle RST$ with vertices $R(1,6)$, $S(1,-6)$, and $T(-3,-6)$; $\triangle UVW$ with vertices $U(-1,0)$, $V(-1,-3)$, and $W(-2,-3)$



4. $\triangle DEF$ with vertices $D(-6,5)$, $E(-6,2)$, and $F(-2,2)$; $\triangle FMW$ with vertices $F(-2,2)$, $M(-2,-4)$, and $W(6,-4)$



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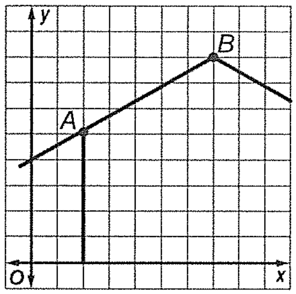
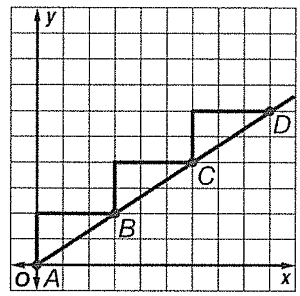
Problem-Solving Practice

Slope and Similar Triangles

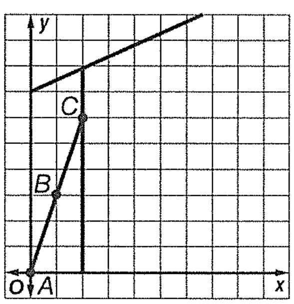
1. The slope of a roof line is also called the pitch. Find the pitch of the roof shown.



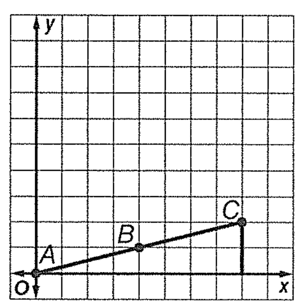
2. A carpenter is building a set of steps for a bunk bed. The plan for the steps is shown below. Using points A and B, find the slope of the line up the steps. Then verify that the slope is the same at a different location by choosing a different set of points.



3. A ladder is leaning up against the side of a house. Use two points to find the slope of the ladder. Then verify that the slope is the same at a different location by choosing a different set of points.



4. The graph shows the plans for a bean bag tossing game. Use two points to find the slope of the game. Then verify that the slope is the same at a different location by choosing a different set of points.



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9.1 Constructing Scatter Plots

A **scatter plot** is a plot on the coordinate plane used to compare two sets of data and look for a correlation between those data sets. An **association** is a relationship or dependence between data. For example, the price of oil and the price of gasoline have a strong association. The daily price of oil and the number of penguins swimming in the ocean on that day most likely have no association at all. However, to find this association we need to make a scatter plot.

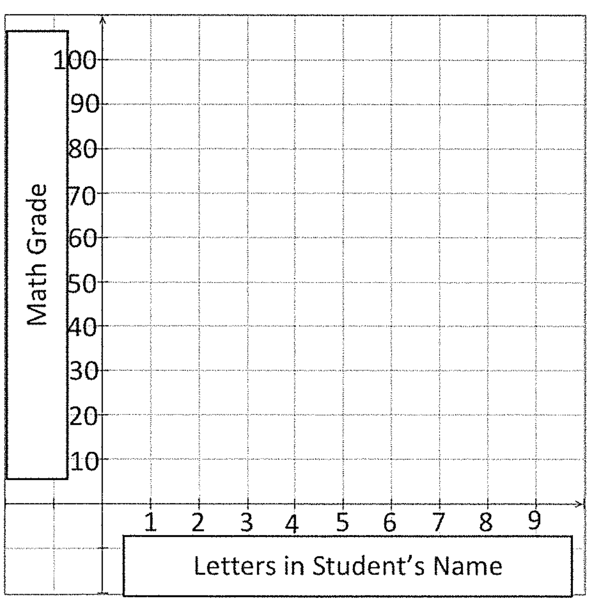
Start with the Data

Before we can make a scatter plot, we need two sets of data that we want to compare. For example, we might compare the number of letters in a student's first name and their math grade. Do people with shorter names tend to score higher in math? Do people with the lowest grades have longer names? These are questions of relationship, or correlation, that we can explore with a scatter plot once have some data. That data set might look like this:

Name	Nichole	Josiah	Kame	Gungar	Roberto	Frank	John	Herman	Sami	Daimon
Letters	7	6	4	6	7	5	4	6	4	6
Grade	58	83	61	70	31	76	81	70	72	57

Name	Yolina	Johanne	Karolinea	Kurt	Addison	Ian	Dennis	Ophelia	Kristina	Bradford
Letters	6	7	9	4	7	3	6	7	8	8
Grade	77	90	87	83	76	78	87	87	80	41

Prepare the Coordinate Plane



Now that we have our data, we need to decide how to put this data on the coordinate plane. We can let the x-axis be the number of letters in a student's name and the y-axis be the students overall math grade. Once we have decided this we should label our axes.

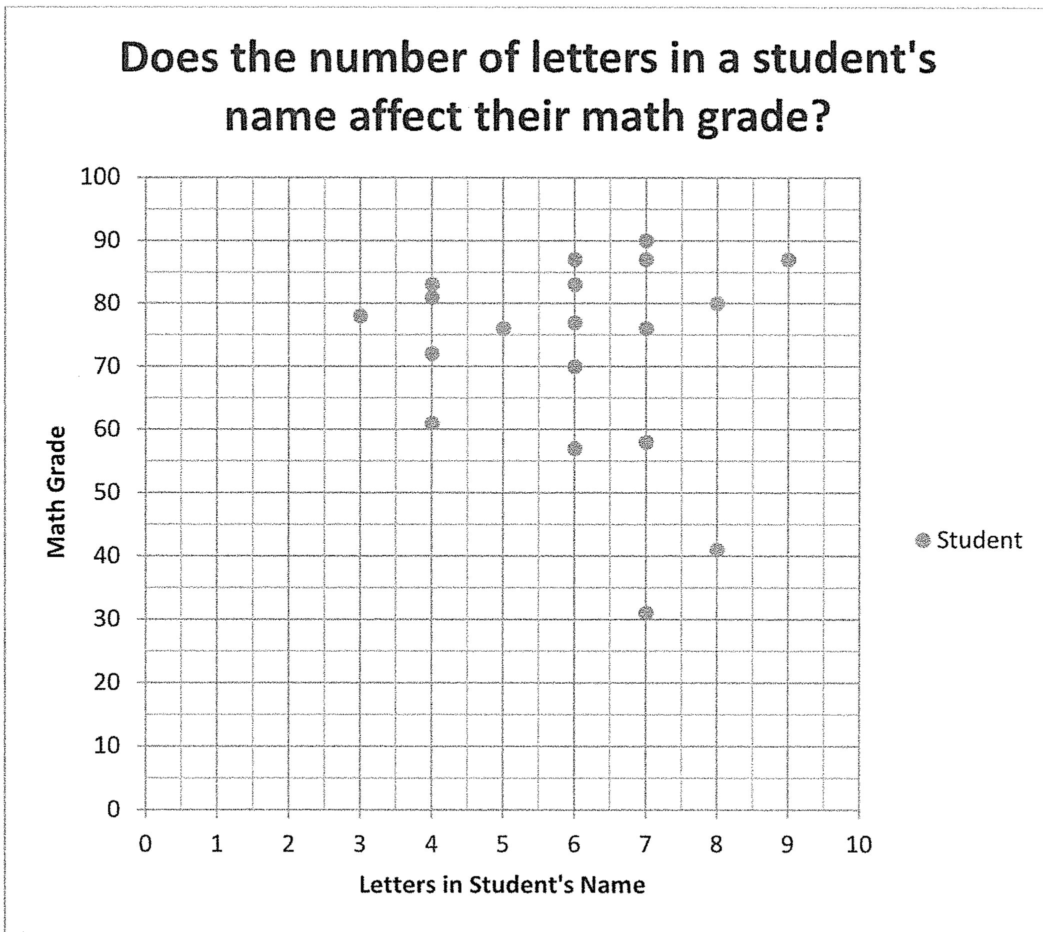
Next we'll need to decide on a scale and interval. The scale is the low to high number on the axis and the interval is what we count by. Notice first of all that we're only looking at Quadrant I because we won't have negative amounts of letters or negative grades. Since the grades can be from zero to one hundred, we might choose to count by tens on the y-axis giving us a scale of 0-100 and an interval of 10. Since the letters range from three to nine, we might count by ones on the x-axis. This gives us a scale of 0-10 with an interval of 1.

When to use a broken axis

A broken axis is useful whenever more than half of the area of the scatter plot will be blank. Nobody likes to see a blank graph with all the data in one tiny area. So instead, we zoom use by using a broken axis. If the range of your data is less than the lowest data point, a broken axis may be useful. For example, in our math test situation above if everyone scored above a 60%, then we might break the y-axis and begin counting at 60. We could then count by 4's to make it up to 100%.

Plot the Points

Finally we would then plot each person on the graph. So Nicholas will be the point $(7, 58)$, Josiah the point $(6, 83)$, and so forth. Using Excel to make our scatter plot, the final scatter plot might look like the following. Notice that each dot on the graph represents a person. While the labeling is not necessary, it may be useful in some circumstances.



Many times on a scatter plot you may have the same data point multiple times. One way to represent this fact is to put another circle around the data point. Let's add a few new students to our data set: Johnathan (9 letters and 87 math score), Jacob (5 letters and 76 math score), and Helga (5 letters and 76 math score). The new graph could look like this:

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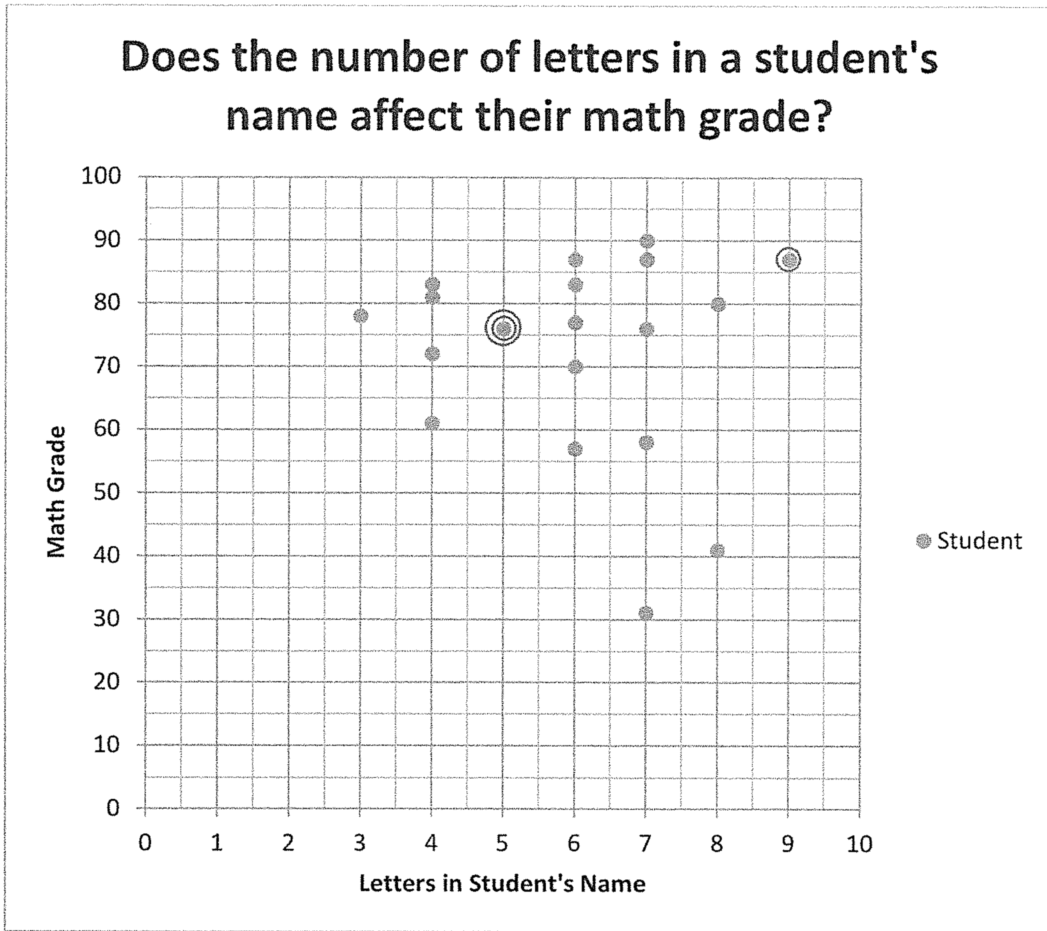

While this practice is not necessarily standard, it can be useful as a visual representation of what is happening with the data. We can more easily see the multiple data points this way. In Excel, you wouldn't get the red circles. Those would have to be put in by hand.

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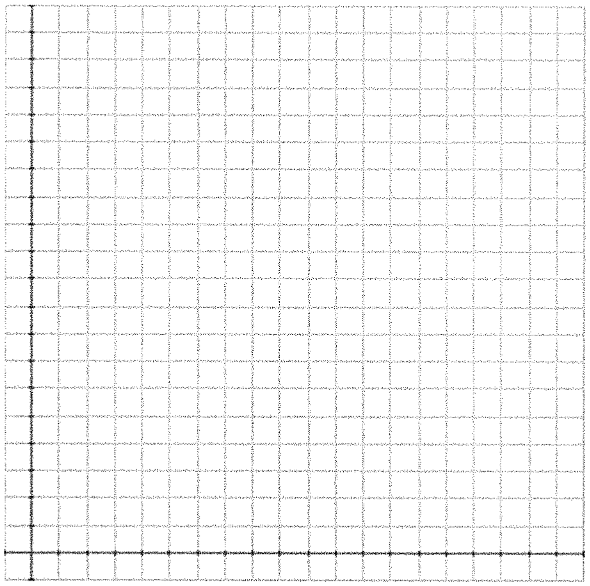
Lesson 9.1

Use the given data to answer the questions and construct the scatter plots.

Pathfinder Character Level vs. Total Experience Points

Level	2	3	6	9	10	11	14	15	17	20
XP	15	35	150	500	710	1050	2950	4250	8500	24000



1. Which variable should be the independent variable (x-axis) and which should be the dependent variable (y-axis)?

2. Should you use a broken axis? Why or why not?

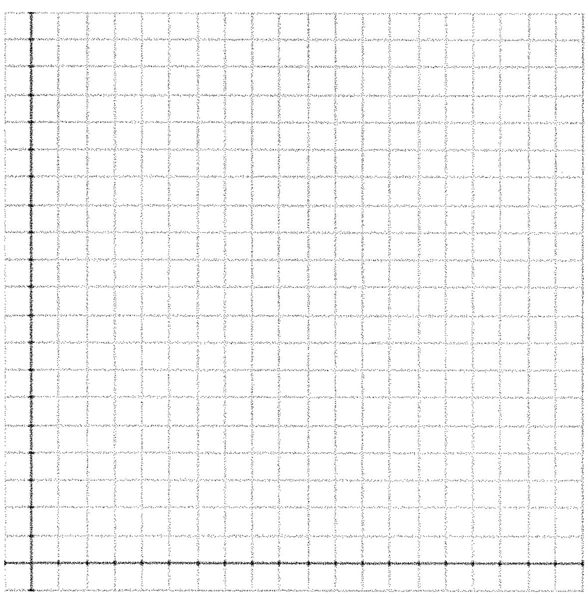
3. What scale and interval should you use for the x-axis?

4. What scale and interval should you use for the y-axis?

5. Construct the scatter plot.

Age vs. Weekly Allowance

Age	12	12	13	13	14	14	15	15	16	16
Allowance	0	5	5	8	10	15	20	20	25	30



6. Which variable should be the independent variable (x-axis) and which should be the dependent variable (y-axis)?

7. Should you use a broken axis? Why or why not?

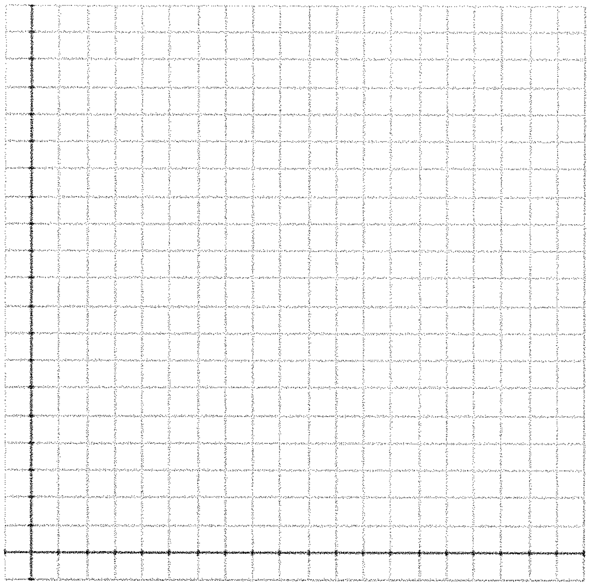
8. What scale and interval should you use for the x-axis?

9. What scale and interval should you use for the y-axis?

10. Construct the scatter plot.

Age vs. Number of Baby Teeth

Age	5	6	7	7	8	9	10	11	11	12
Baby Teeth	20	19	17	15	10	10	8	4	2	2



11. Which variable should be the independent variable (x-axis) and which should be the dependent variable (y-axis)?

12. Should you use a broken axis? Why or why not?

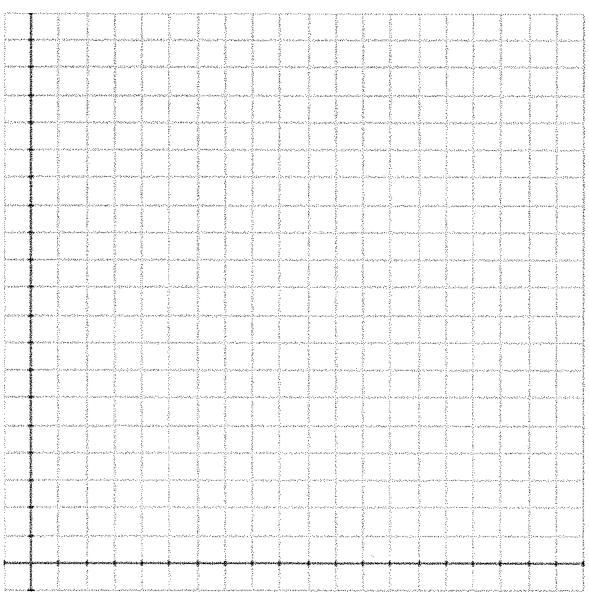
13. What scale and interval should you use for the x-axis?

14. What scale and interval should you use for the y-axis?

15. Construct the scatter plot.

Car Speed (in mph) vs. Gas Mileage (in mpg)

Speed	20	25	35	40	45	55	65	80	90	100
Mileage	25	27	28	30	31	32	30	29	25	22



16. Which variable should be the independent variable (x-axis) and which should be the dependent variable (y-axis)?

17. Should you use a broken axis? Why or why not?

18. What scale and interval should you use for the x-axis?

19. What scale and interval should you use for the y-axis?

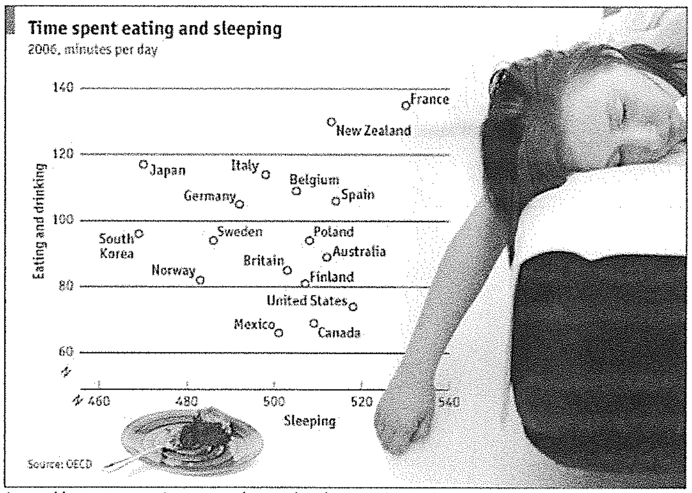
20. Construct the scatter plot.

9.2 Analyzing Scatter Plots

Now that we know how to draw scatter plots, we need to know how to interpret them. A scatter plot graph can give us lots of important information about how data sets are related if we understand what each part of the graph means.

Reading Data Points

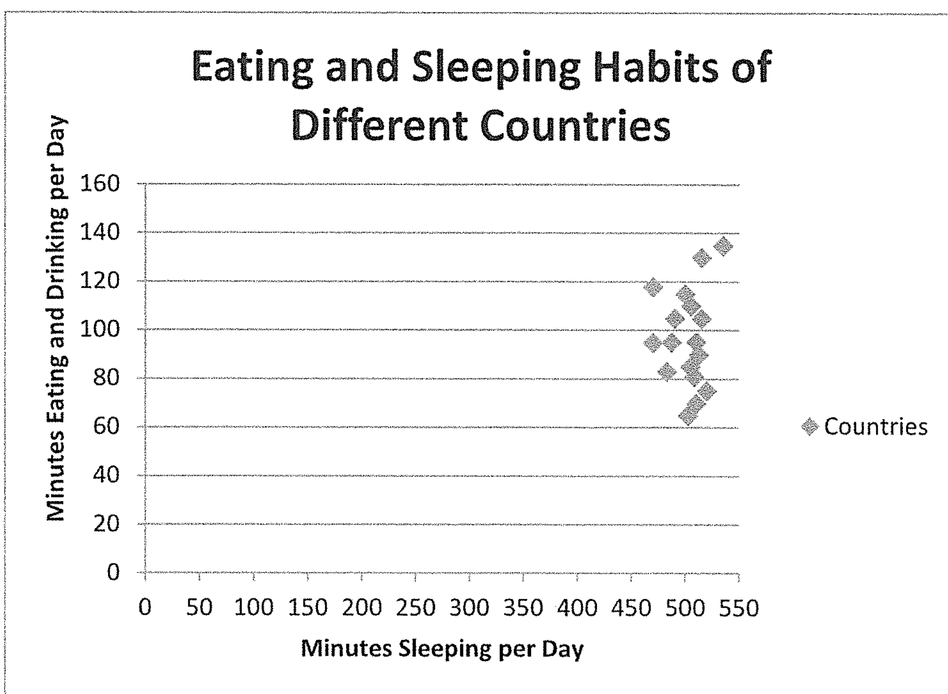
Each individual point on a scatter plot represents a single idea. For example, in the picture below each point represents a country. The axes tell us information about that country. The y-axis tells us about how many minutes per day that country spends eating and drinking. The x-axis tells us about how many minutes per day that country spends sleeping. Can you find the United States on this scatter plot? About how many minutes do we sleep per day? About how many minutes we spend eating and drinking per day? Are these numbers reasonable to you?



http://www.visualquest.in/2010/09/severalsimple-and-very-useful.html

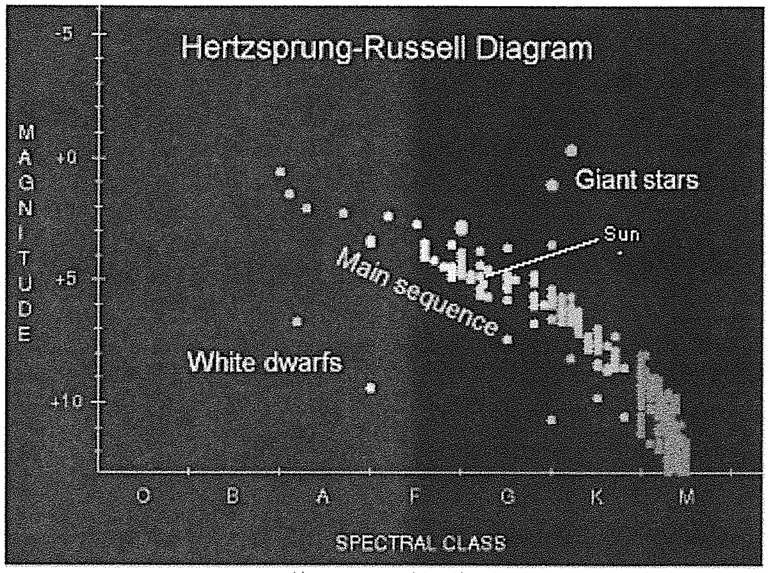
Another thing to notice about this scatter plot is that it uses the broken axis symbol (that little Z looking thing). This means that they don't start counting from zero on either axis. They skip ahead to a reasonable starting point but still apply a scale after that point. Even with the broken axis they must count by something in each direction. In this case, they count by 20 minutes on the x-axis and the y-axis as well.

If we did not use the broken axis, it might look more like the scatter plot below. To be able to label the data points, it is useful in this case to use the broken axes.



Outliers

An outlier is a data point that is significantly far away from the majority of the data. There is no precise mathematical definition for what makes a data point an outlier. It's usually somewhat obvious. For example, notice that White Dwarf Stars and Giant Stars are both outliers in the below scatter plot showing a star's spectral class (temperature) versus its magnitude (brightness).

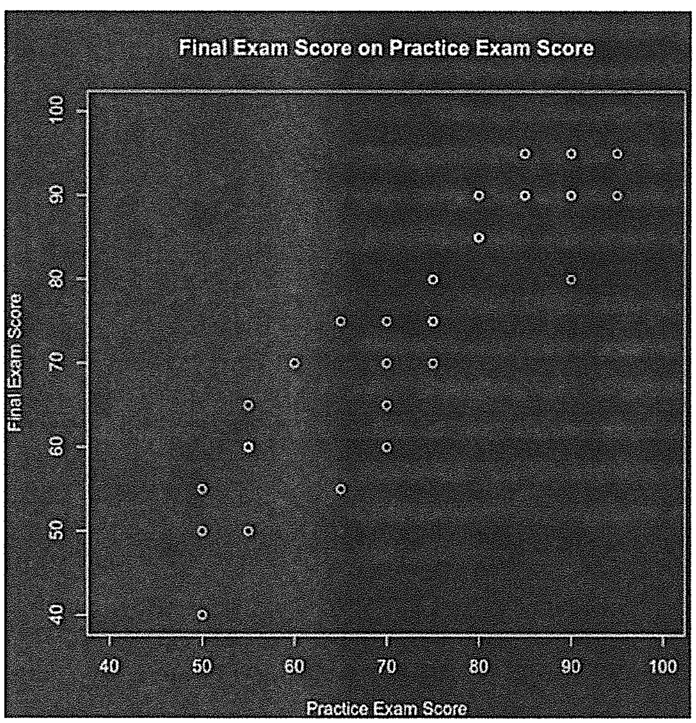


http://starplot.org/docs/ch1.html

Why do we care about outliers? We care because outliers often throw off the analysis of the data set. For example, let's say you have three test grades in math class: 80%, 80%, and 80%. Your current class average is, you guessed it, 80%. However, if we throw in an outlier, like a 0%, for the next test, your class average drops down to 60%. You have dropped two letter grades from a B- to a D-. Yikes! The outlier sure hurt your grade.

Positive and Negative Associations

An **association**, sometimes called a correlation, is a relationship between two data sets. For example, in the above star scatter plot, there appears to be a relationship between a star's temperature and brightness. We'd have to know more about the science of stars to fully interpret the graph, but we can see there is an association because most of the data follows a pattern (except for those pesky outliers).



http://www.r-bloggers.com/r-tutorial-series-basic-polynomial-regression/

In fact, the more tightly clumped the data is, the stronger the association is. We might say that there is a strong association between the brightness and temperature of a star. In the scatter plot to the left, we see a slightly weaker association between scores on a practice exam and scores of the final exam.

We would also say that the scatter plot to the left has a positive association because it appears that the students who scored higher on the practice exam also scored higher on the final exam. As one variable (practice exam score) increased, the other variable (final exam score) also increased. We call this a positive association.

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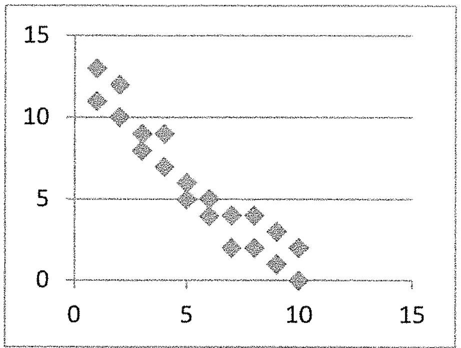
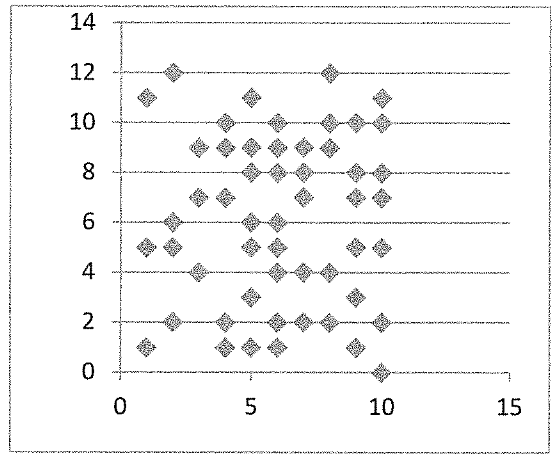


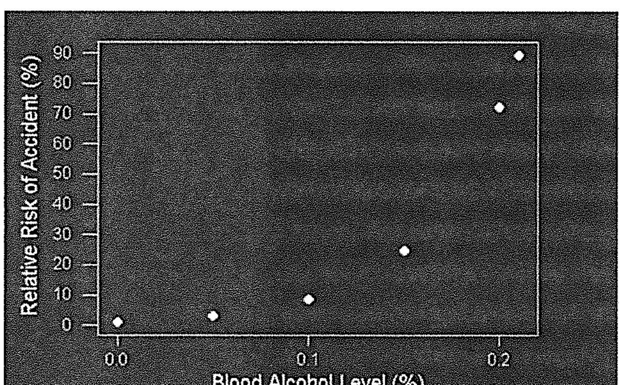
There are also negative associations. These associations are recognized by the fact that as one variable increases, the other decreases. For example, as the supply of oil increases, the cost of gasoline decreases. They have a negative association. A scatter plot with a negative association might look like the graph to the left.

No association would mean that there appears to be no relationship between the two data sets (or variables). For example, we might consider the daily price of tea and the daily number of fruit flies born. There is likely no relationship between those two things which would produce a graph similar to the one to the right.



Linear or Non-Linear Associations

Whether the association is positive or negative, it may appear linear or non-linear. A linear association would be a scatter plot where the data points clump together around what appears to be a line. The negative association graph above and to the left is an example of a linear association. The scatter plot about practice and final exams is an example of a positive linear association.

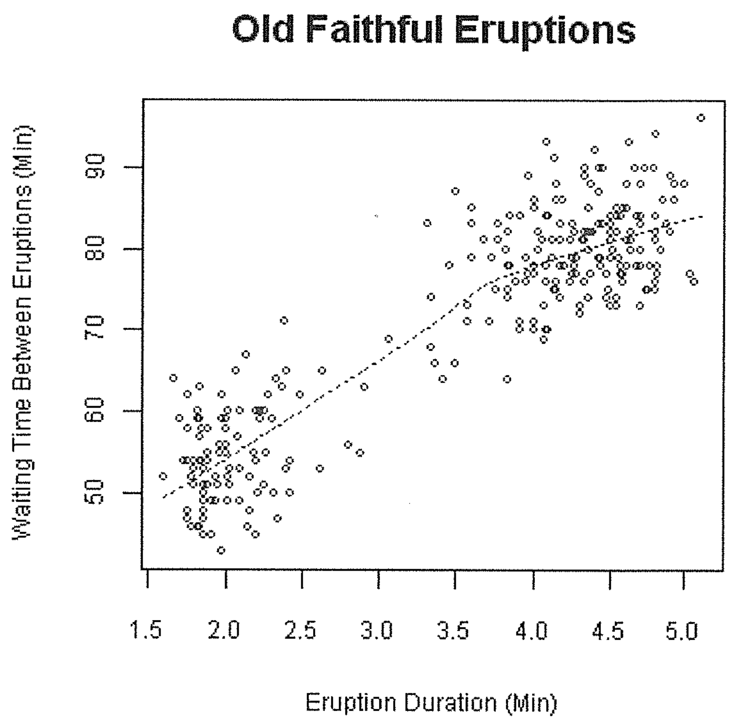


http://wps.prenhall.com/esm_walpole_probstats_7/55/14203/3635978.cw/content/index.html

A non-linear association is usually curved to some extent. There are many types of curves that it could fit, but we'll just focus on the fact that it doesn't look a line and therefore is non-linear. Consider the graph to the left showing the relative risk of an accident compared to the blood alcohol level. As you can see, the graph curves sharply up when there is more alcohol in the blood stream. This should not only serve as an example of non-linear scatter plot, but also the risks of drinking and driving.

Clustering

Clustering is when there is an association, but it appears to come in clumps. Consider the following scatter plot that shows the time between eruptions and eruption duration of Old Faithful. Notice how the points cluster towards the lower left and upper right. While this does show us a positive association (meaning the longer between eruptions, the longer the next eruption will last), it also shows us that there are not very many medium length eruptions. They are either short eruptions with short wait times or long eruptions with long wait times.



http://en.wikipedia.org/wiki/Scatter_plot

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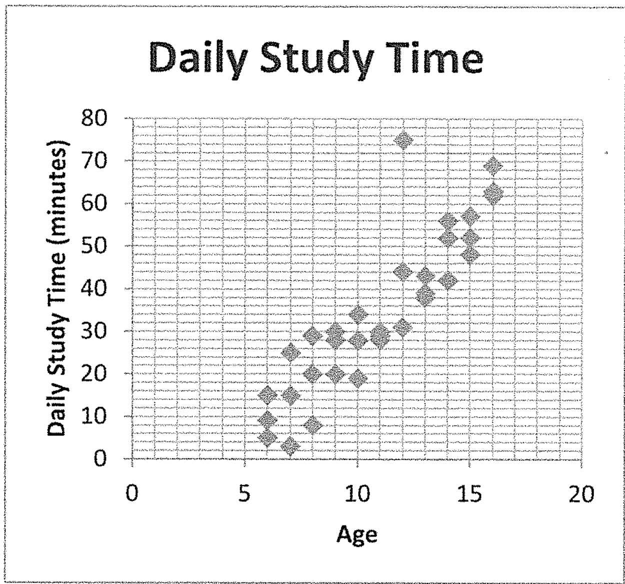
Lesson 9.2

Use the given scatter plots to answer the questions.

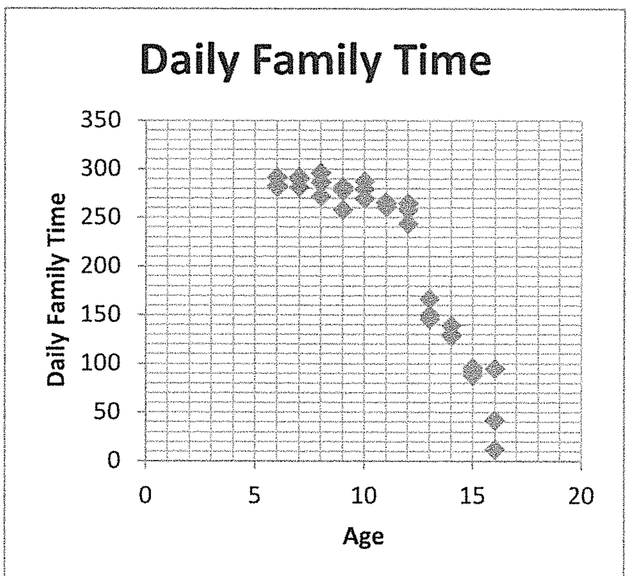


1. Does this scatter plot show a positive association, negative association, or no association? Explain why.

2. Is there an outlier in this data set? If so, approximately how old is the outlier and about how many minutes does he or she study per day?

3. Is this association linear or non-linear? Explain why.

4. What can you say about the relationship between your age and the amount that you study?



5. Does this scatter plot show a positive association, negative association, or no association? Explain why.

6. Is there an outlier in this data set? If so, approximately how old is the outlier and about how many minutes does he or she spend with family per day?

7. Is this association linear or non-linear? Explain why.

8. What can you say about the relationship between your age and the amount of time that you spend with family?

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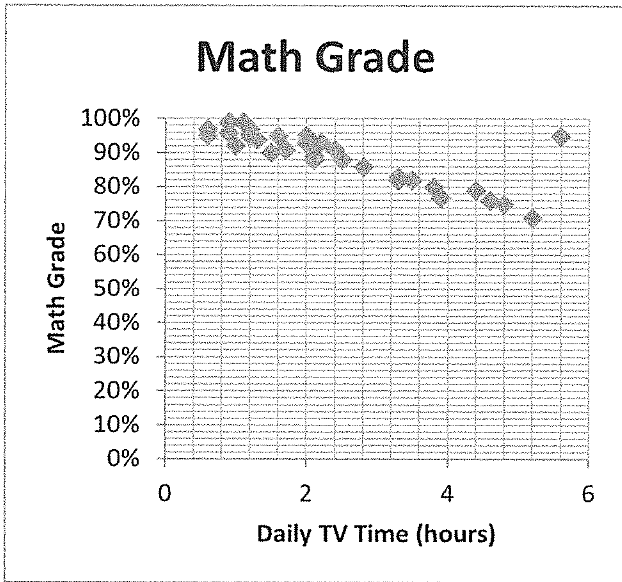
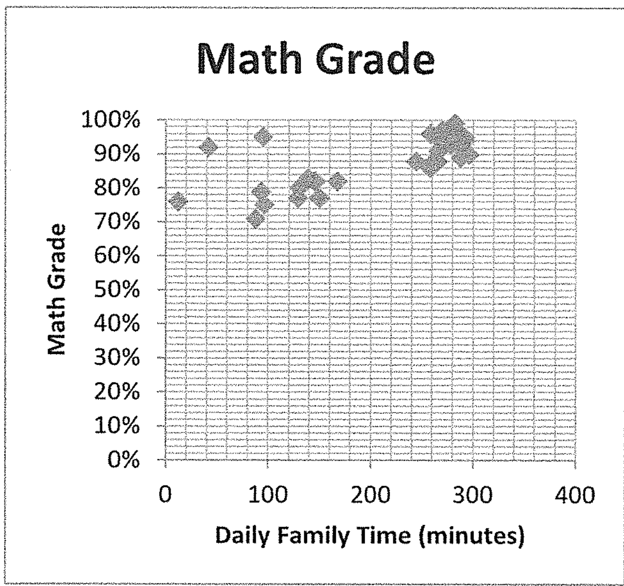


9. Does this scatter plot show a positive association, negative association, or no association? Explain why.

10. Is there an outlier in this data set? If so, approximately how much does that person watch TV daily and what is his or her approximate math grade?

11. Is this association linear or non-linear? Explain why.

12. What can you say about the relationship between the amount of time you watch TV and your math grade?



13. Does this scatter plot show a positive association, negative association, or no association? Explain why.

14. Is there an outlier(s) in this data set? If so, approximately how much time does that person(s) spend with his or her family daily and what is his or her approximate math grade?

15. Is this association linear or non-linear? Explain why.

16. What can you say about the relationship between the amount of time that you spend with your family and your math grade?

17. Are there any other patterns that you notice in this data?

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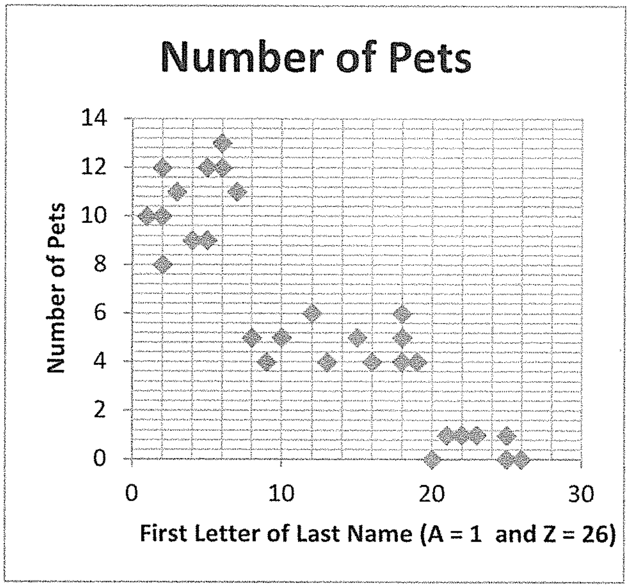


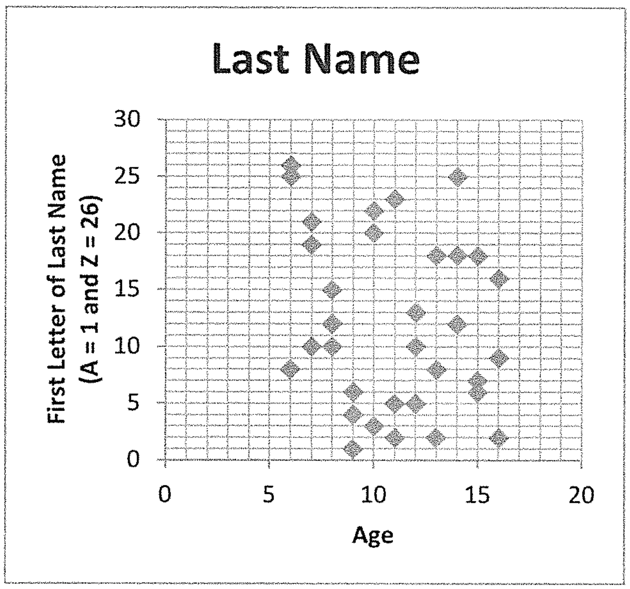
18. Does this scatter plot show a positive association, negative association, or no association? Explain why.

19. Is there an outlier(s) in this data set? If so, approximately how many pets does that person(s) have?

20. Is this association linear or non-linear? Explain why.

21. What can you say about the relationship between your last name and the number of pets you have?

22. Are there other patterns that you notice about people's last names and how many pets they have?



23. Does this scatter plot show a positive association, negative association, or no association? Explain why.

24. Is there an outlier(s) in this data set? If so, approximately how old is that person?

25. Is this association linear or non-linear? Explain why.

26. What can you say about the relationship between your last name and your age?

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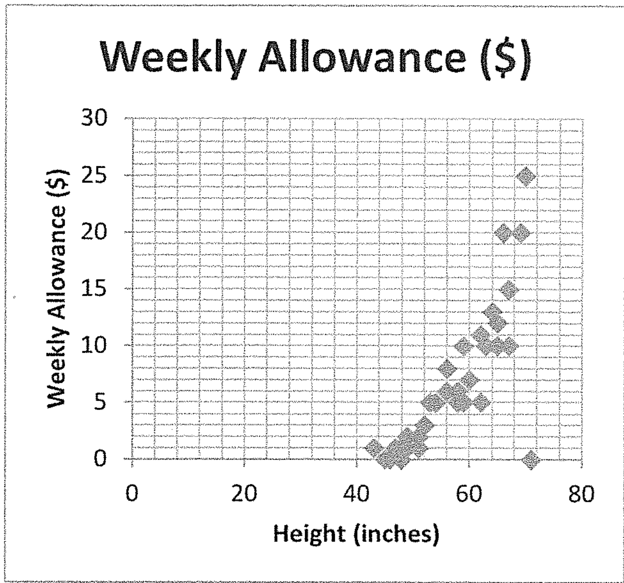


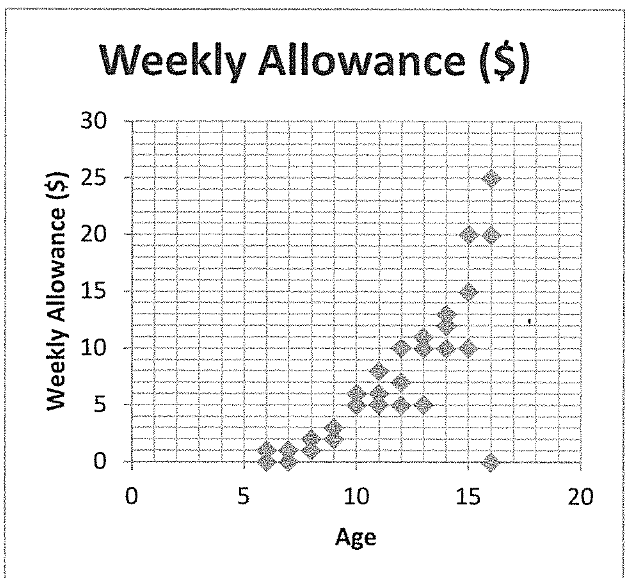
27. Does this scatter plot show a positive association, negative association, or no association? Explain why.

28. Is there an outlier(s) in this data set? If so, approximately how tall is that person and how much does he or she make in allowance each week?

29. Is this association linear or non-linear? Explain why.

30. What can you say about the relationship between your height and your allowance?

31. Do you think that being taller means that you will get more allowance? In other words, do you think this relationship is a causation or a correlation?



32. Does this scatter plot show a positive association, negative association, or no association? Explain why.

33. Is there an outlier(s) in this data set? If so, approximately how old is that person and how much does he or she make in allowance each week?

34. Is this association linear or non-linear? Explain why.

35. What can you say about the relationship between your age and your allowance?

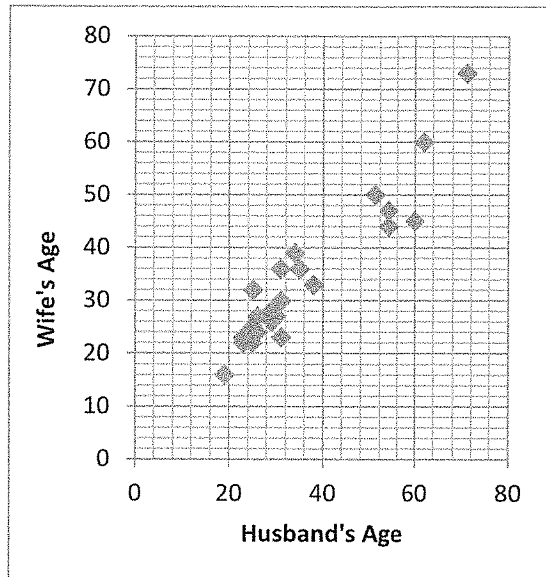
36. Do you think that being older means that you will get more allowance? In other words, do think this relationship is a causation or a correlation?

9.3 The Line of Best Fit

When we have a scatter plot that suggests a linear association, it is often useful to draw in a line of best fit to help us interpret the data more accurately. A **line of best fit** is a line drawn on the scatter plot such that the distance between each of the points and the line are minimized. Let's look at some examples.

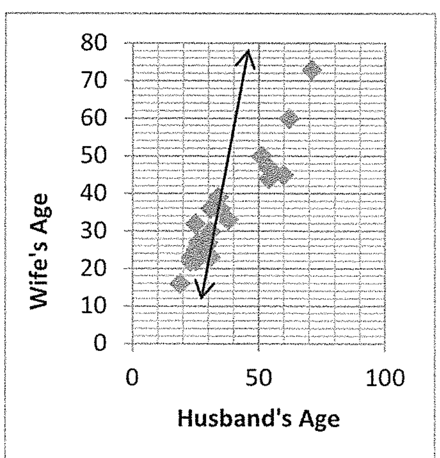
Drawing the Line of Best Fit

Finding the true line of best fit is quite an involved task if we do it by hand. While programs like Excel will automatically draw in the line of best fit for us, for now we will focus on informally drawing a line of best fit. In other words, we know that our line is not the exact line of best fit, but it will be a nice estimate. Consider the scatter plot to the right.

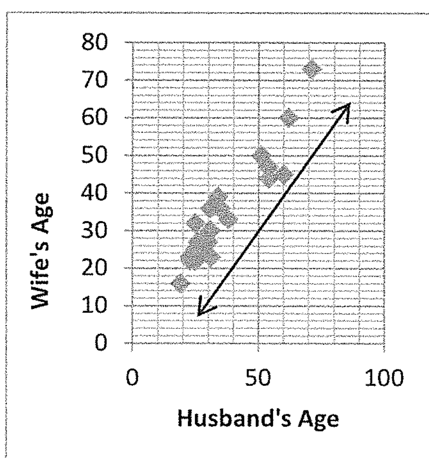


In this scatter plot there are 24 couples represented and it appears that there is a positive linear association between their ages. Generally speaking it looks like the older the husband is, the older the wife is. If we wanted to informally draw a line of best fit in this scatter plot, we would look for a place where we the line would roughly split data in half and have the same general rate of change (or slope) as the data.

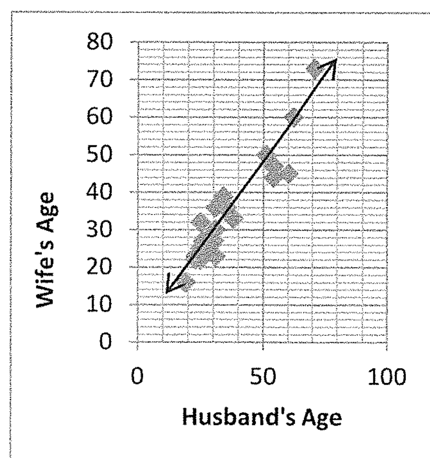
Now consider the three scatter plots below. Which line of best fit seems most appropriate? The first attempted line of best fit does appear to cut the data roughly in half, but it definitely doesn't match the rate of change that the data seems to represent. The second attempted line of best fit seems to match the rate of change but doesn't roughly cut the data in half. The third one is our best option for an informal line of best fit.



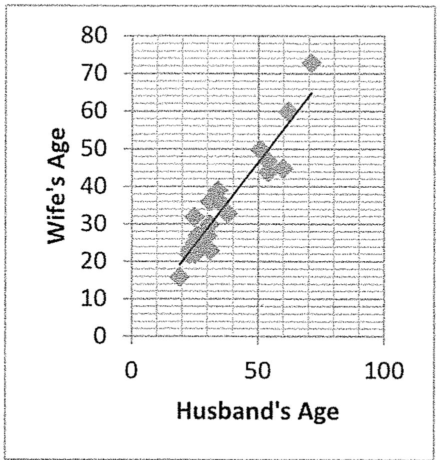
First Attempt



Second Attempt



Third Attempt

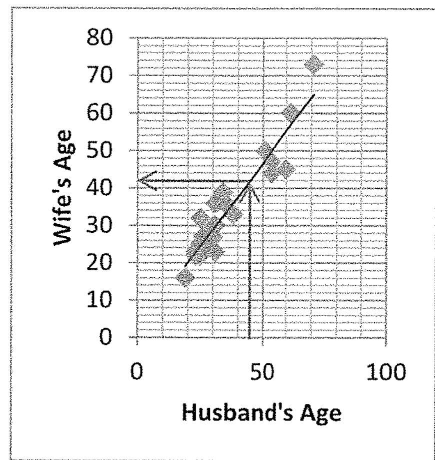


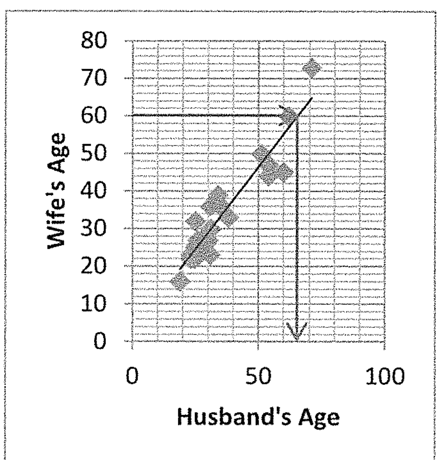
Now for the sake of comparison, let's see the actual line of best fit that Excel comes up with. It looks like our line of best fit is very close to the true line of best fit.

Before drawing in the line of best fit on a given data set, it may be useful to lay down a pen or pencil on the scatter plot and try to arrange the pen where the line of best fit should be. Once you have visualized where the line of best fit should be, then draw it in.

Extrapolating with the Line of Best Fit

To **extrapolate** means to estimate or predict an answer in an unknown situation. We can use the line of best fit to make these predictions from the data. For example, using the above line of best fit, how old would we expect the wife to be of a husband that was 45 years old? We don't have a data point there, so we don't know what the answer to this would be, but we can extrapolate using the line of best fit. Go to 45 years old on the husband axis and go up to the line of best fit. Note that the line of best fit is at a height of 42 years old for the wife meaning this would be a good estimate for how old we would expect the wife to be.





If the wife were 60 years old, how old would we expect the husband to be? This time go to the height of 60 on the wife axis and travel over to the line of best fit. It appears to be at about 65 years old on the husband axis, so we would expect the husband to be near that age.

Notice that these are only estimates and would not necessarily be exactly what we would find in real life, but it is useful as a guideline.

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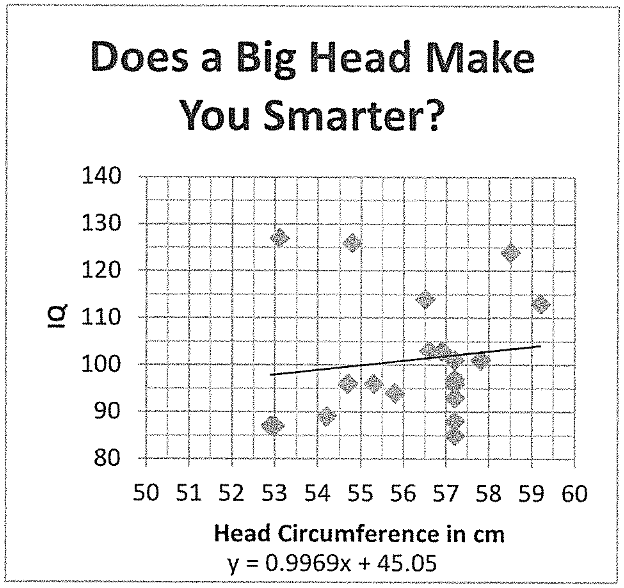
The Equation of the Line of Best Fit

Since we have a *line* of best fit, we know that a line can be expressed as an equation. In fact, we are most familiar with the slope intercept form of an equation. We can use this line to extrapolate data further, learn more about the rate of change, and more. Let's look at some data taken from sets of twins where they were studying if there was an association between the size of a person's skull and his or her IQ.



First of all, notice that all the data is clustered between the 50 cm and 60 cm mark, so Excel decided it would be beneficial to use a broken axis on this graph. Secondly, notice that Excel has drawn in the line of best fit and given us the equation for that line.

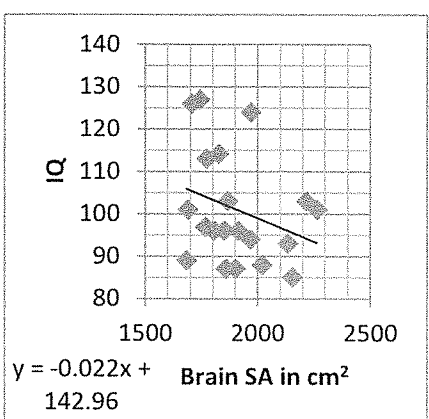
At first glance it appears that there may be no association between the size of your skull and your IQ. The line of best fit is nearly flat suggesting either a constant association or no association at all. However, because of the broken axes, this is misleading.

Let's first approximate the equation for ease of analysis. The slope of 0.9969 is very close 1 and the y-intercept is very close to 45, so let's approximate the line of best fit to be $y = x + 45$.

What does the slope mean in this context? The slope is approximately 1, which means that for every one centimeter increase in skull size we would expect a one point increase in IQ. So maybe there is something to that old "egg head" comment, as mean as it is.

What does the y-intercept mean in this context? The y-intercept is about 45, which tells us that no matter the size of a person's head, their IQ is very unlikely to drop below 45. Even a skull size of zero centimeters in circumference would supposedly have an IQ of 45, but we know this isn't possible.

What would be the expected IQ if a person had a head circumference of 80 cm? In our equation, the y represents the IQ and x represents the head circumference. Simply plug in and solve like this: $y = 80 + 45 = 125$ to see that the expected IQ would be about 125. If a person had an IQ of 150, what would we expect their head circumference to be using our line of best fit? $150 = x + 45$ and then subtract 45 from both sides to see that $x = 115\ cm$. That's a big head!



This final graph shows the surface area of the brain compared to IQ. If we rounded the slope and intercept, the equation of the line of best fit is approximately $y = -\frac{1}{50}x + 143$. This means that the IQ goes down one for every additional 50 cm^2 in surface area. In context that means that the more of your brain that is "exposed", the lower your IQ.

Data Used

Brain data came from: http://lib.stat.cmu.edu/datasets/IQ_Brain_Size

Husband and wife data came from: http://www.statcrunch.com/5.0/viewreport.php?reportid=10183

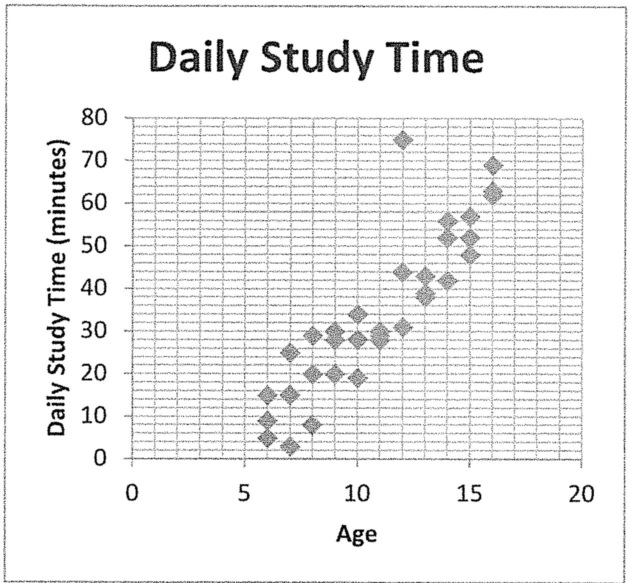
Head Circumference	Brain Surface Area	Brain Volume	Body Weight	IQ
54.7	1913.88	1005	57.607	96
54.2	1684.89	963	58.968	89
53	1902.36	1035	64.184	87
52.9	1860.24	1027	58.514	87
57.8	2264.25	1281	63.958	101
56.9	2216.4	1272	61.69	103
56.6	1866.99	1051	133.358	103
55.3	1850.64	1079	107.503	96
53.1	1743.04	1034	62.143	127
54.8	1709.3	1070	83.009	126
57.2	1689.6	1173	61.236	101
57.2	1806.31	1079	61.236	96
57.2	2136.37	1067	83.916	93
57.2	2018.92	1104	79.38	88
55.8	1966.81	1347	97.524	94
57.2	2154.67	1439	99.792	85
57.2	1767.56	1029	81.648	97
56.5	1827.92	1100	88.452	114
59.2	1773.83	1204	79.38	113
58.5	1971.63	1160	72.576	124

Couple	Husband's Age	Wife's Age
1	25	22
2	25	32
3	51	50
4	25	25
5	38	33
6	30	27
7	60	45
8	54	47
9	31	30
10	54	44
11	23	23
12	34	39
13	25	24
14	23	22
15	19	16
16	71	73
17	26	27
18	31	36
19	26	24
20	62	60
21	29	26
22	31	23
23	29	28

Lesson 9.3

Draw an informal line of best fit on the given scatter plot and explain why you drew the line where you did.

1.



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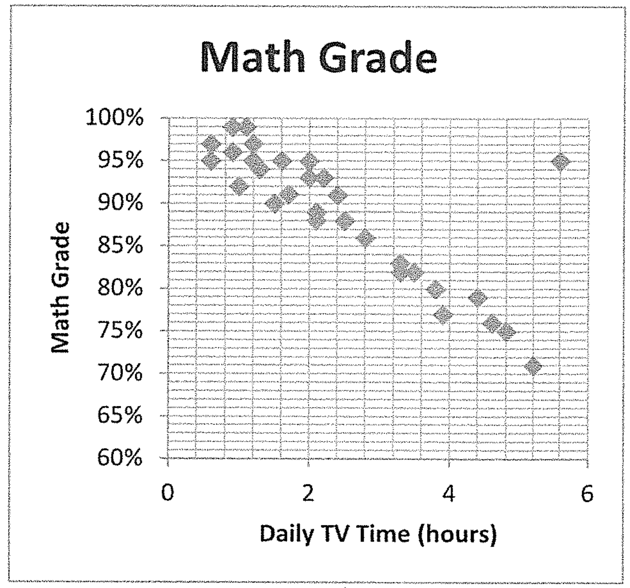

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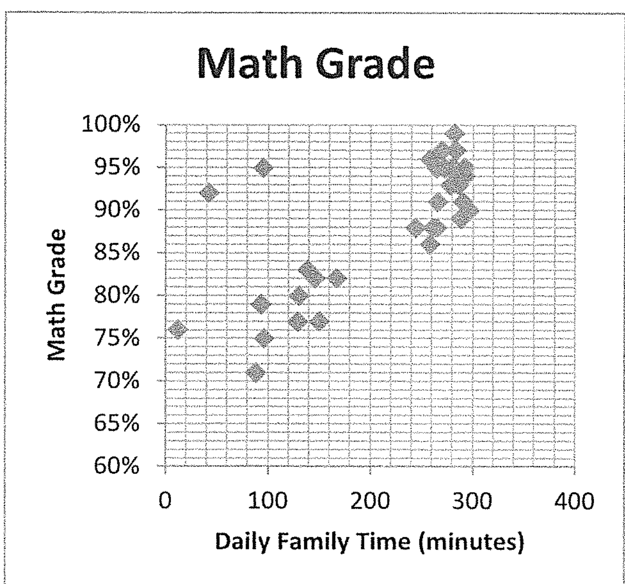

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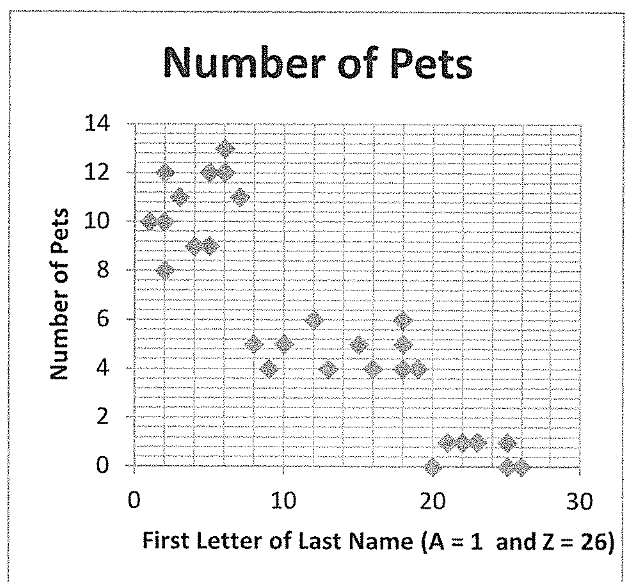

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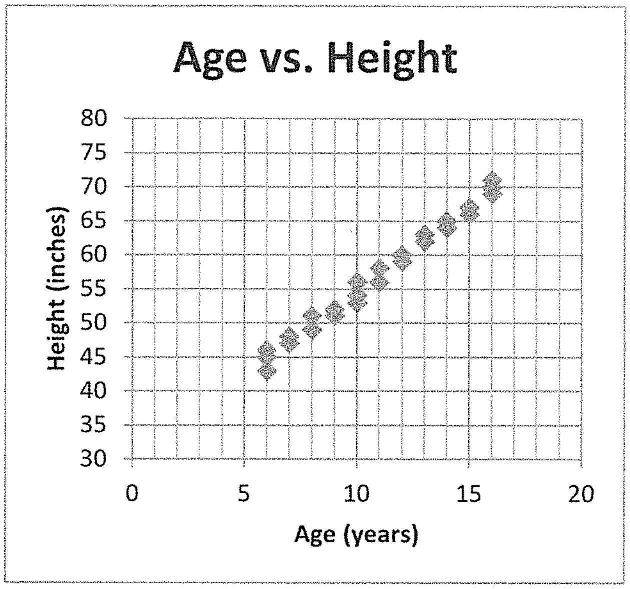
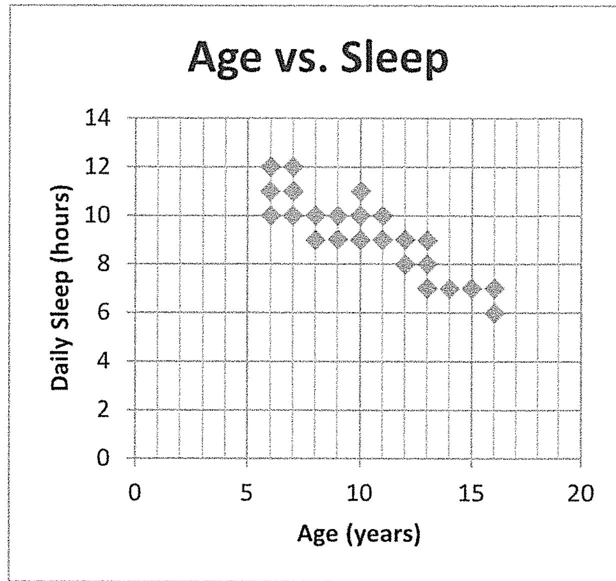
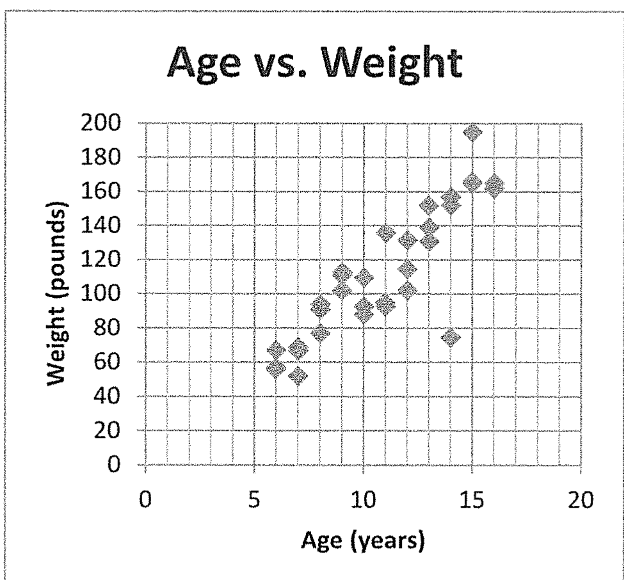
5.


Age vs. Height
Height (inches)
80
75
70
65
60
55
50
45
40
35
30
0
5
10
15
20
Age (years)

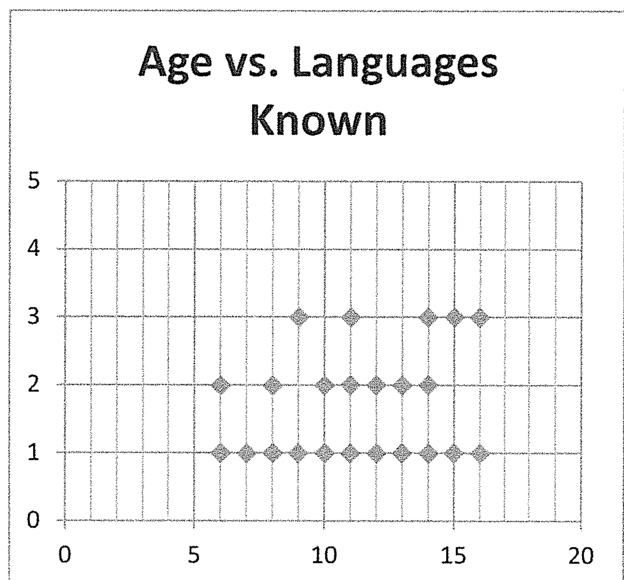
6.


Age vs. Sleep
Daily Sleep (hours)
14
12
10
8
6
4
2
0
0
5
10
15
20
Age (years)

7.


Age vs. Weight
Weight (pounds)
200
180
160
140
120
100
80
60
40
20
0
0
5
10
15
20
Age (years)

8.


Age vs. Languages Known
5
4
3
2
1
0
0
5
10
15
20

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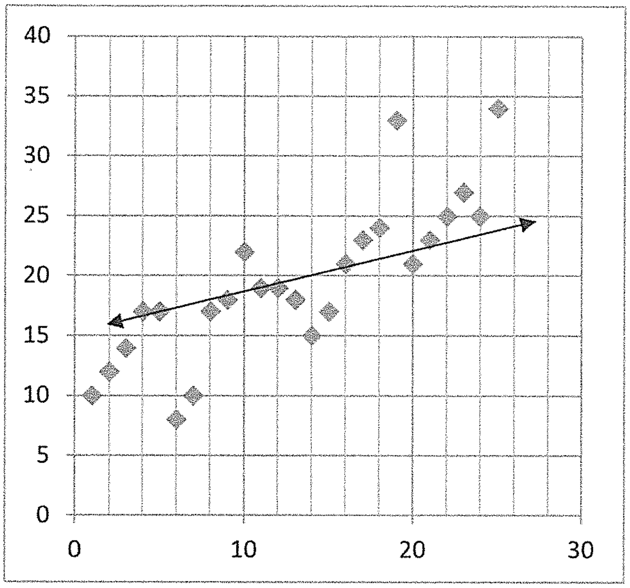
Determine whether the drawn line of best fit is accurate or not. Explain why you think your position is true.

9.



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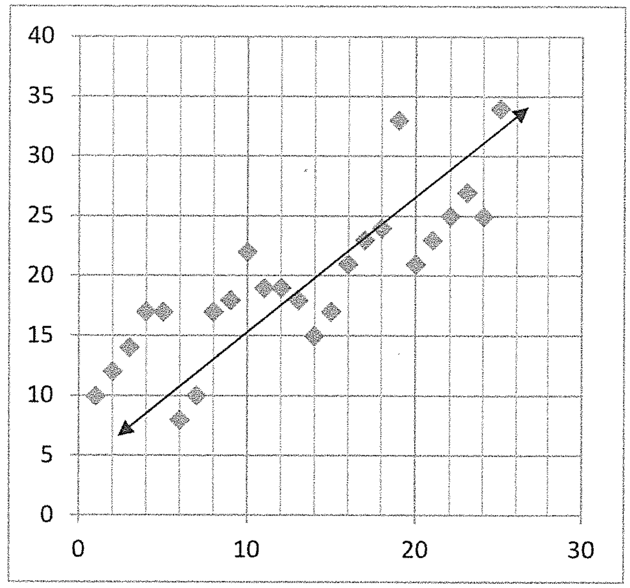
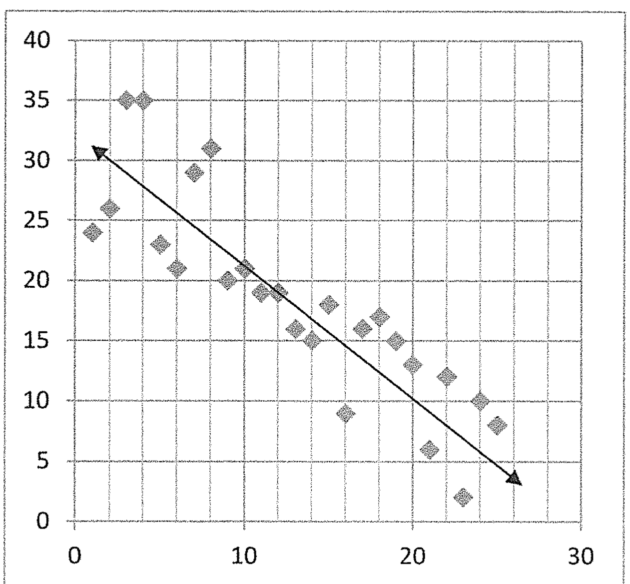
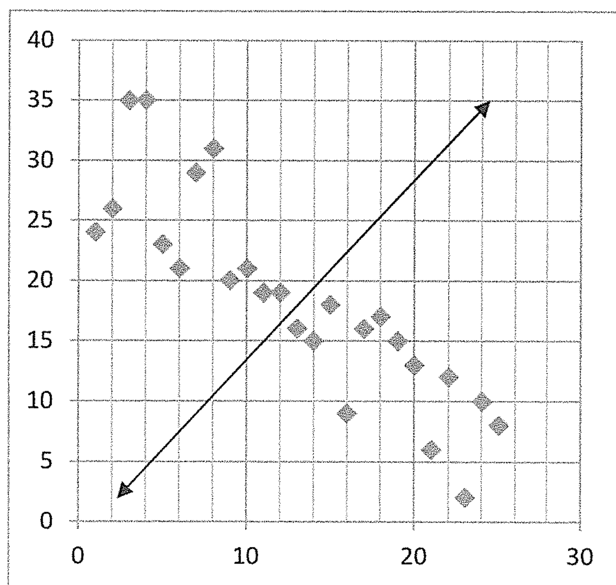


11.



12.



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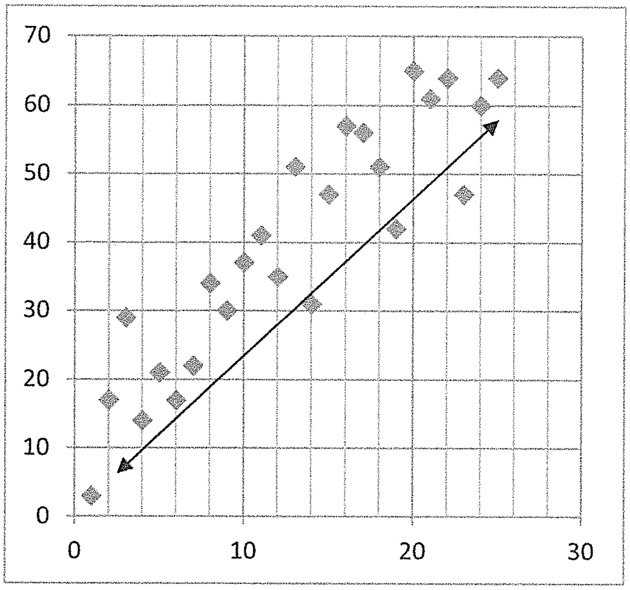
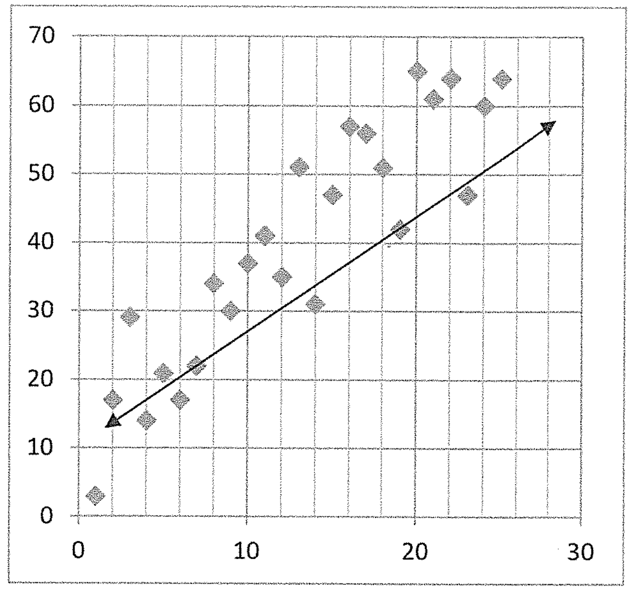
13.



70
60
50
40
30
20
10
0
0
10
20
30

14.



70
60
50
40
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20
10
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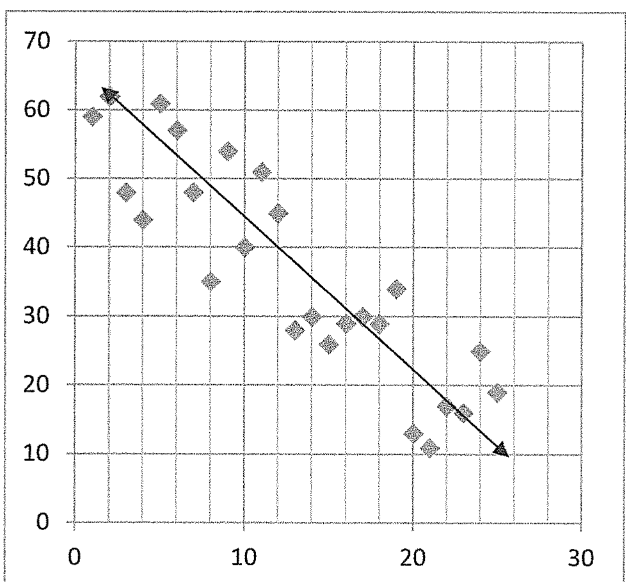
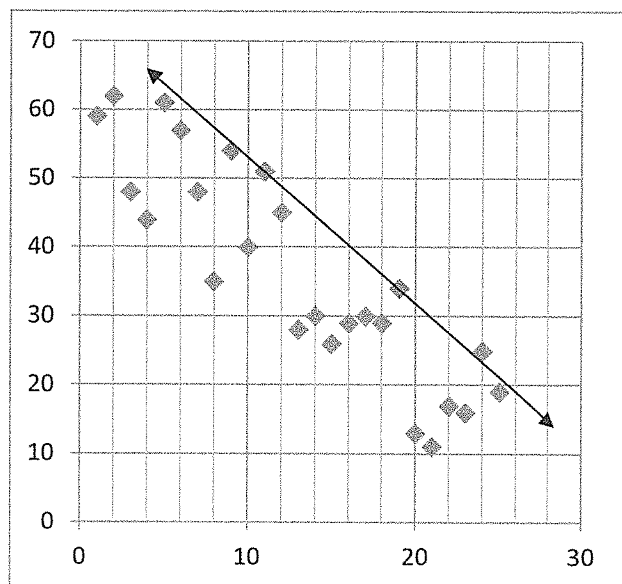
30

15.



70
60
50
40
30
20
10
0
0
10
20
30

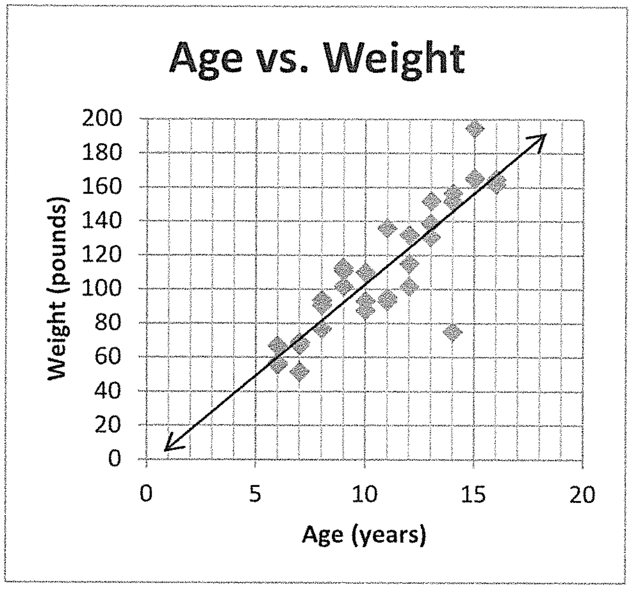
16.



70
60
50
40
30
20
10
0
0
10
20
30

60

Use the given line of best fit or equation of the line of best fit to answer the following questions.



17. Using the graph only, about how much would you expect an 18 year old to weigh?

18. Using the graph only, about how much would you expect a 4 year old to weigh?

19. Using the graph only, if a person weighed 80 pounds, how old would you expect them to be?

20. Using the graph only, if a person weighed 120 pounds, how old would you expect them to be?

The line of best fit for the scatter plot showing age (x-value) compared to weight (y-value) is approximately:

$$y = \frac{21}{2}x - \frac{7}{2}$$

21. Using the line of best fit equation (show your work), about how much would you expect an 18 year old to weigh? How does this answer compare to the answer you gave to problem number 17?

22. Using the line of best fit equation (show your work), about how much would you expect an 4 year old to weigh? How does this answer compare to the answer you gave to problem number 18?

23. Using the line of best fit equation (show your work), about how old would you expect a person to be if they weighed 80 pounds? How does this answer compare to the answer you gave to problem number 19?

24. Using the line of best fit equation (show your work), about how old would you expect a person to be if they weighed 120 pounds? How does this answer compare to the answer you gave to problem number 20?

25. What is the rate of change (slope) of the line of best fit? What does the slope represent in this context and does that make sense?

26. What is the initial value (y-intercept) of the line of best fit? What does it represent in this context and does that make sense?

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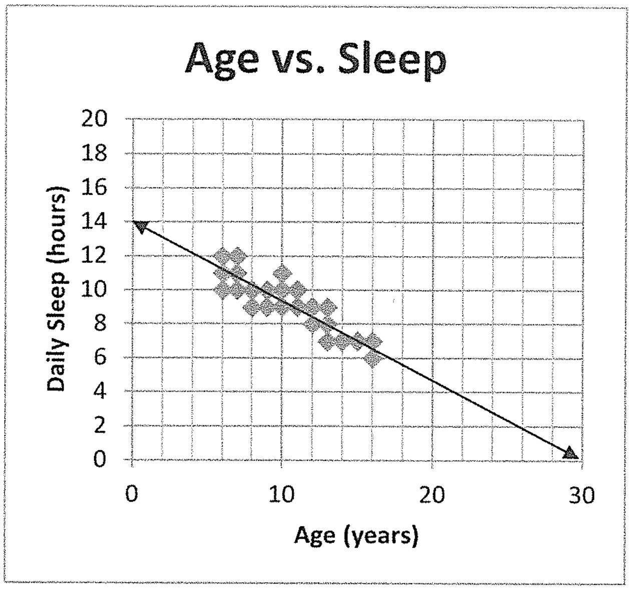


27. Using the graph only, about how much would you expect a 22 year old to sleep?

28. Using the graph only, about how much would you expect a 4 year old to sleep?

29. Using the graph only, if a person slept 6 hours, how old would you expect them to be?

30. Using the graph only, if a person slept 13 hours, how old would you expect them to be?

The line of best fit for the scatter plot showing age (x-value) compared to daily hours of sleep (y-value) is approximately:

$$y = -\frac{1}{2}x + 14$$

31. Using the line of best fit equation (show your work), about how much would you expect a 22 year old to sleep? How does this answer compare to the answer you gave to problem number 27?

32. Using the line of best fit equation (show your work), about how much would you expect a 4 year old to sleep? How does this answer compare to the answer you gave to problem number 28?

33. Using the line of best fit equation (show your work), about how old would you expect a person to be if they slept 6 hours? How does this answer compare to the answer you gave to problem number 29?

34. Using the line of best fit equation (show your work), about how old would you expect a person to be if they slept 13 hours? How does this answer compare to the answer you gave to problem number 30?

35. What is the rate of change (slope) of the line of best fit? What does the slope represent in this context and does that make sense?

36. What is the initial value (y-intercept) of the line of best fit? What does it represent in this context and does that make sense?

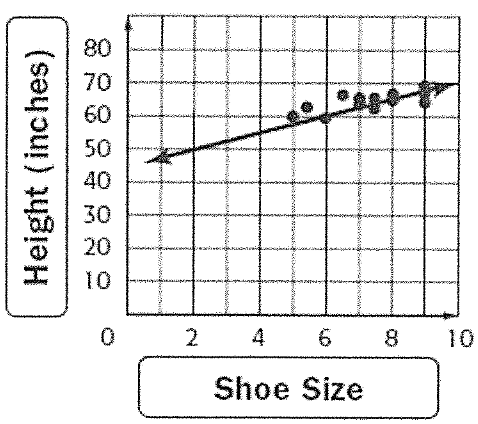
Scatter Plots and Data Analysis

Lines of Best Fit
Pages681 and 683

The results of a survey about women's shoe sizes and heights are shown.

Height (inches) and Shoe Size			
Shoe Size	Height	Shoe Size	Height
8	66	$6\frac{1}{2}$	65
8	65	9	68
$7\frac{1}{2}$	65	$7\frac{1}{2}$	63
7	62	7	64
7	62	$5\frac{1}{2}$	62
9	68	5	60
9	65	9	67
9	65	6	59

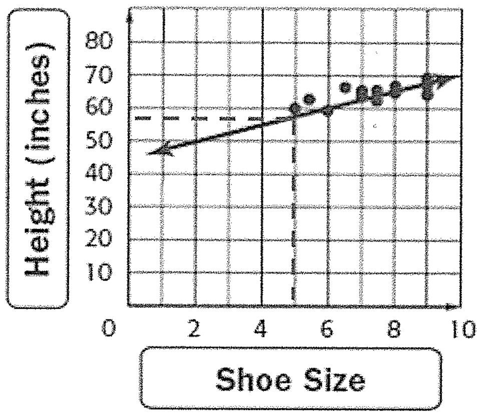
a.Construct a scatter plot of the data. Then draw and assess a line that best represents the data.
Let the horizontal axis, or x-axis represent the shoe size. Let the vertical axis, or y-axis represent height. Then graph the ordered pairs (shoes size, height). Draw a line that fits the data.



The data points are either on the line of best fit or very close to the line, so the line of best fit is a good model of the data.

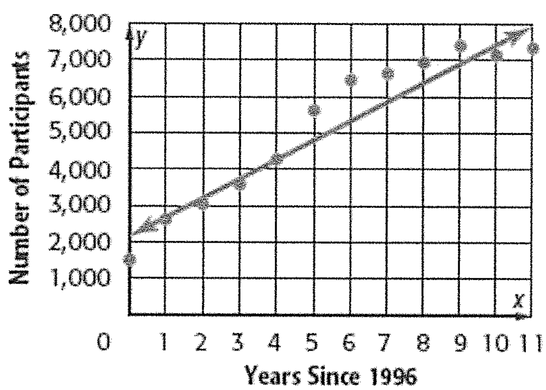
b.Use the line of best fit to make a conjecture about the height of a female who wears a size 5 shoe.

Draw a vertical line from the x-value of 5 to the line of best fit. Then draw a horizontal line from that point to the y-axis.



The line intersects the y-axis about three-fourths of the way between 50 and 60, so the height of a female who wears a size 5 shoe is about 57.5 inches.

The scatter plot shows the number of girls who participate in ice hockey.



a. Write an equation in slope-intercept form for the line of best fit that is drawn, and interpret the slope and y-intercept.

Determine the slope of the line by choosing two points that the line passes through.Use (2.5, 3,500) and (3.5, 4,000).

$$m = \frac{4,000 - 3,500}{3.5 - 2.5}$$
$$= 500$$

Then use the slope and a point on the line to find the y-intercept.

$y = mx + b$	Slope-intercept form
$3,500 = 500(2.5) + b$	Use the point (2.5, 3,500). Replace x with 2.5, y with 3,500, and m with 500.
$2,250 = b$	Solve for b.

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A sample equation for the line of best fit is $y = 500x + 2{,}250$. Every year an additional 500 girls play ice hockey. In 1996, 2,250 girls played ice hockey.

b.Use the equation to make a conjecture about the number of girls that will participate in ice hockey in 2020.

In 2020, x will equal 2020 − 1996 or 24. Evaluate the equationfor $x = 24$.

$y = 500(24) + 2{,}250$ Substitute.

$y = 14{,}250$ Simplify.

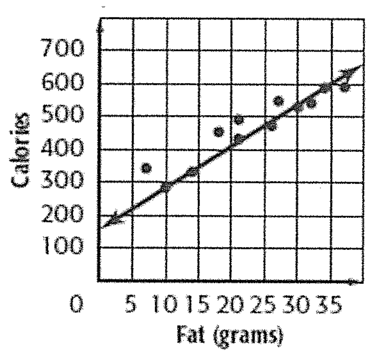
There will be approximately 14,250 girlsparticipating in ice hockey in 2020.

Model with Mathematics The table shows fat and Calories for fast food sandwiches.

Fat (grams)	21	10	14	21	30	34	32	37	27	26	18	7
Calories	490	280	330	430	530	590	540	590	550	470	450	340

a. Construct a scatter plot of the data. Draw and assess a line that best represents the data.

Let the horizontal axis, or x-axis represent fat. Let the vertical axis, or y-axis represent Calories. Then graph the ordered pairs (fat, Calories). Draw a line that fits the data.



Since the data points are all close to the line, the line of best fit drawn is a good model of the data.

b. Write an equation in slope-intercept form for the line of best fit, and interpret the slope and y-intercept.

Determine the slope of the line by choosing two points that the line passes through.Use (10, 280) and (14, 330).

$$m = \frac{330 - 280}{14 - 10}$$

$$= 12.5$$

Then use the slope and a point on the line to find the y-intercept.

$y = mx + b$	Slope-intercept form
$280 = 12.5(10) + b$	Use the point (10, 280). Replace x with 10, y with 280, and m with 12.5.
$155 = b$	Solve for b.

A sample equation for the line of best fit is $y = 12.5x + 155$. A 1 gram increase in fat increases the Calories by 12.5 grams. A sandwich with 0 grams of fat would be 155 Calories.

c. Use the equation to make a conjecture about the number of grams of fat in a sandwich with 350 Calories.

Evaluate the equationfor $y = 350$.

$350 = 12.5x + 155$	Replace y with 350.
$195 = 12.5x$	Subtract 155 from each side.
$\frac{195}{12.5} = \frac{12.5x}{12.5}$	Divide each side by 12.5.
$15.6 = x$	Simplify.

There will be approximately 15.6 grams of fat in a sandwich with 350 Calories.

ule

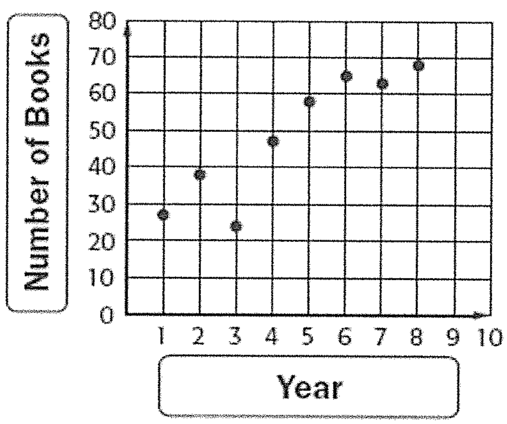
Scatter Plots and Data Analysis

Scatter Plots
Pages 671 and 673

1 Construct a scatter plot of the number of books donated over time.

Year	1	2	3	4	5	6	7	8
Number of Books	27	38	24	47	58	65	63	68

Let the horizontal axis, or x-axis represent the year. Let the vertical axis, or y-axis represent the number of books. Then graph the ordered pairs (year, number of books).



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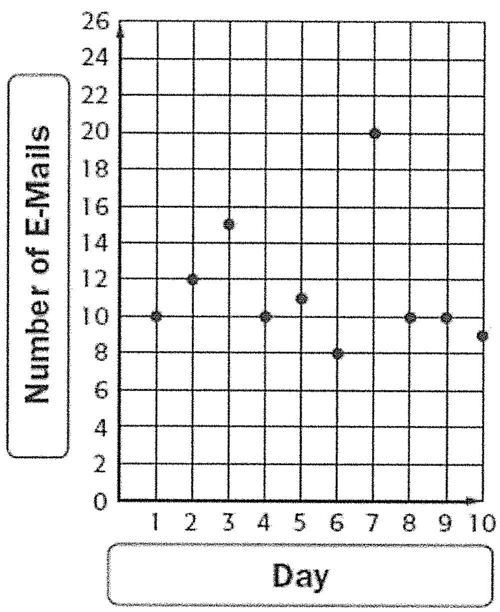
9 **The table shows the number of junk E-mails Petra received over the last 10 days.**

Day	1	2	3	4	5	6	7	8	9	10
Number of E-Mails	10	12	15	10	11	8	20	10	10	9

a. Construct a scatter plot of the data.

Let the horizontal axis, or *x*-axis represent the day. Let the vertical axis, or *y*-axis represent the number of E-mails. Then graph the ordered pairs (day, number of E-mails).



b. Interpret the scatter plot of the data based on the shape of the distribution.

Consider the different associations and patterns.

Variable Association: There does not appear to be any relationship between the variables.

Linear Association: Linearity cannot be determined.

Other Patterns: There are no clusters. There appears to be an outlier at 20 E-mails.

c. If a relationship exists, make a conjecture about the number of junk E-mails on Day 15.

Since the scatter plot shows no association between the data, it is not possible to predict how many E-mails will be received on Day 15.

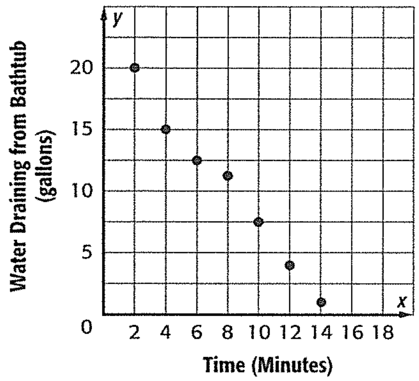
NAME ______________________ DATE ____________ PERIOD ________

Skills Practice

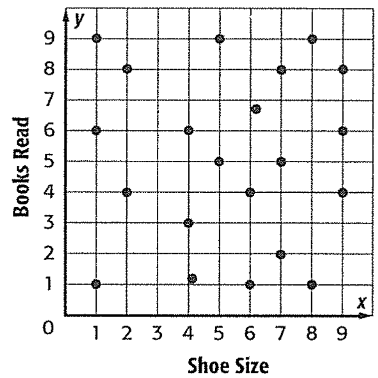
Scatter Plots

Explain whether the scatter plot of the data for each of the following shows a *positive*, *negative*, or *no* association. Interpret the scatter plot.

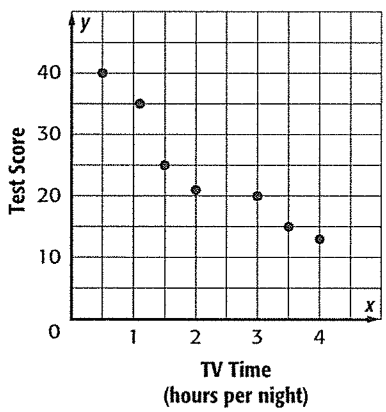
1.



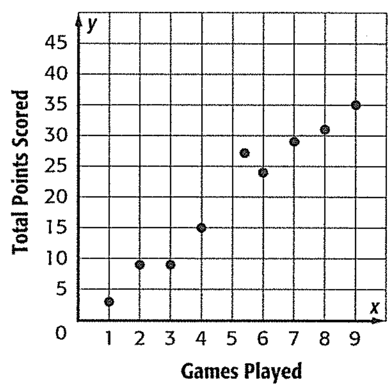
2.



3.



4.



5. E-MAIL Construct a scatter plot of the number of E-mails Vincent received over the past six days. Interpret the scatter plot.

Day	1	2	3	4	5	6
E-mails	16	21	3	11	19	5